Analysis of Lagged Weight Windows for Implicit Monte Carlo Variance Reduction

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Radiation Transport Equation

- The Implicit Monte Carlo (IMC) software utilized for this analysis solves the following radiative transport equation coupled to a material energy equation

$$\frac{1}{c}\frac{\partial I}{\partial t} + \vec{\Omega} \cdot \nabla I = -\sigma \left(I + \frac{ac}{4\pi} T^4 \right)$$

$$\frac{\partial e_m}{\partial t} = \sigma c \left(e_m - a T^4 \right)$$

- The results we present here are only for gray (frequency-integrated) problems, so there is no energy dependence

Weight Windows Background

- The method known as Weight Windows is a widely used variance reduction technique applied to Monte Carlo simulations
- A particular objective of this technique is to increase the amount of contributing particles to specific regions that contain low responses, allowing for more accurate and detailed information
- Recent work by Becker and Larsen [2009] developed a theory to analyze how different weight windows approaches will perform on a particular problem

Weight Windows Background

- Becker and Larson studied the Global Flux Weight Windows (GFWW) approach and the Forward-Weighted Consistent Adjoint-Driven Importance Sampling (FW-CADIS) approach
- Their work showed that the following expression can be used to analyze the impact of the specified weight window center, $w(x, E)$, throughout space and energy

$$\phi(\mathrm{x}, E) = Cw(x, E)M(x, E)$$

- For problems where one is interested in the solution everywhere, it is usually desirable that $M(x, E)$, the Monte Carlo Particle Flux, is as uniform as possible
- Wollaber [2008] used the solution of a quasi-diffusion problem to select the weight window center for IMC solutions to radiative transfer problems with a large degree of success

Lagged Weight Windows Correlation

- We used a much simpler approach, which has the benefit of a nearly free improvement in solution statistics
- For a given time step in an IMC calculation we use the previous time step's estimate of the scalar flux for the weight window center

$$w(\mathrm{x}, E, t_n) = \phi(\mathrm{x}, E, t_{n-1})$$

- Using this weight window center with the expression developed by Becker and Larson, the Theoretical Monte Carlo particle flux can be expressed as

$$M(\mathrm{x}, E, t_n) = \frac{\phi(\mathrm{x}, E, t_n)}{C\phi(\mathrm{x}, E, t_{n-1})}$$

Weight Window Boundaries

- The upper limit for the weight window is defined as

$$U = \frac{2\phi(\mathrm{x}, E, t_{n-1})}{1 - w}$$

- The lower limit for the weight window is defined as

$$L = w * U$$

- w is defined as the weight window width factor
- For many of our simulations w was set to 0.2 resulting in a ratio of 5 for the splitting weight to the roulette weight; that is particles with a weight greater than 2.5 times the window center are split and particles with a weight less than 1/2 the window center undergo Russian roulette

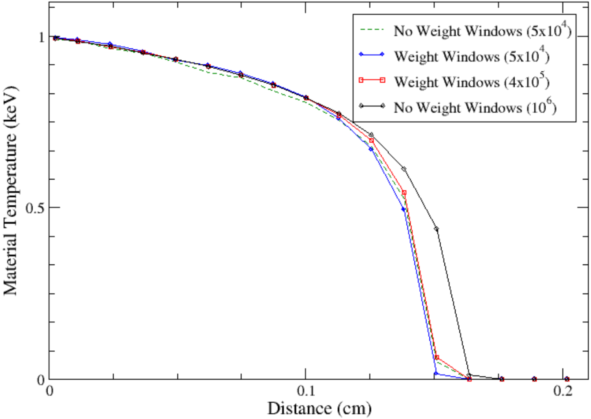
Initial Expectations

- In problems that are optically thick, we expect to obtain a nearly uniform Monte Carlo Particle Flux
  - We expect the scalar flux to change slowly as a function of time
  - Therefore between time steps the ratio between the current time step solution and the previous solution should be close to unity
- In problems that are optically thin (large amounts of streaming), the ratio could be very large near the wave front where $\phi(x, E, t_n) \gg \phi(x, E, t_{n-1})$
- Indeed, we see both of these phenomena in real simulations

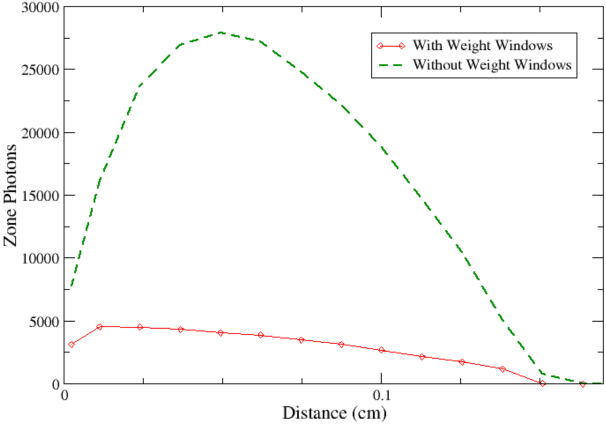
1-D Marshak Wave Configuration

- opacity of $\sigma = 300\,T^{-3}$ cm^{-1} with T in keV
- The initial temperature is 1 eV and there is a boundary source at 1 keV on the left of the problem
- This problem is optically thick everywhere with any reasonable mesh

1-D Marshak Wave Material Temperature



1-D Marshak Wave Particle Flux



Effectiveness of this technique

- The solution obtained using the lagged weight windows approach is at least as accurate as, if not more accurate than, the solution obtained without
- The region behind the wavefront is much noisier in the no weight windows solution than in the weight windows solution
- In the 5×10^4 particles per step simulation, the solution obtained without weight windows took 5203 seconds, while the weight windows solution only took 881 seconds, 1/6th the original runtime.
- The solution with weight windows shows a much more uniform distribution of particles than the solution without

Figure of Merit Calculations

- The Figure of Merit (FOM) used for these simulations is defined by

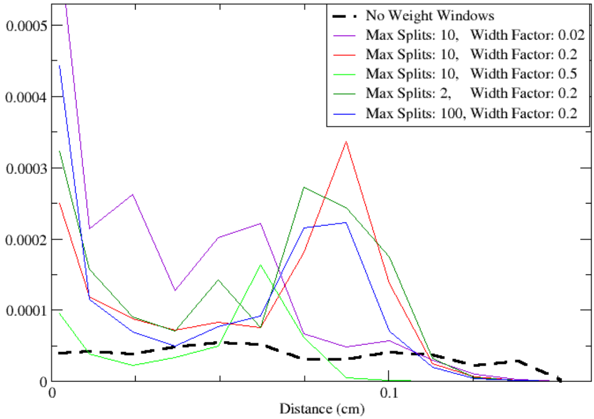
$$FOM = \frac{1}{\sigma^2 t}$$

- σ^2 was determined by taking the variance of an error vector, $\vec{e}$, defined as

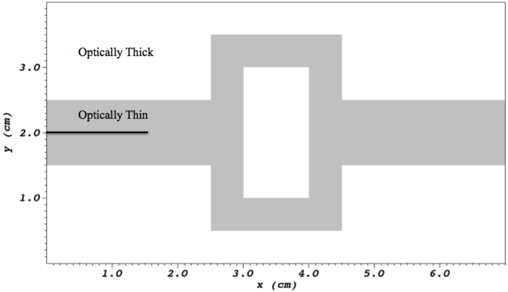
$$\vec{e} = (|T_{\text{converged}} - T_{\text{actual}}|_1, \ldots, |T_{\text{converged}} - T_{\text{actual}}|_n)^T$$

- n is the number of runs performed using different random seeds
- t is the average run time for the n different simulations
- T is the material temperature
- The solution obtained seeding $1\text{x}10^6$ particles without weight windows was considered the converged solution

Marshak Wave Figure of Merit



2-D Crooked Pipe Configuration

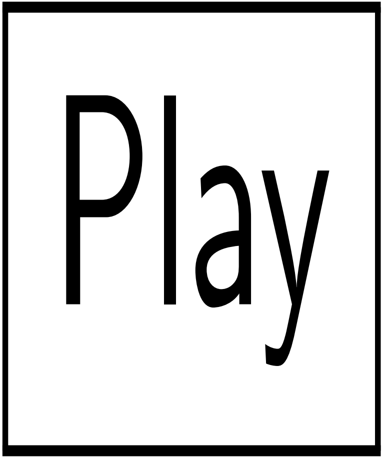


- This configuration is composed of two different types of materials
- There is a 1 keV blackbody source at the $x = 0$ plane

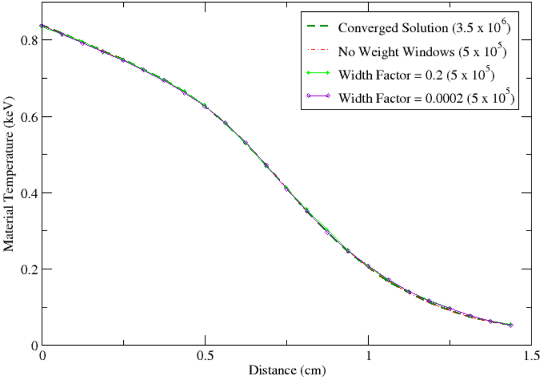
Crooked Pipe Characteristics

- Compared to the material used during the Marshak Wave simulation, the opacity of the optically thin portion of the crooked pipe simulation is much smaller
- In the optically thin material, particles can stream through many spatial zones during a time step
- This leads to a large change in the scalar flux at each time step near the wave front
  - Therefore the simulation is expected to experience spikes in the particle flux towards the particle wave front

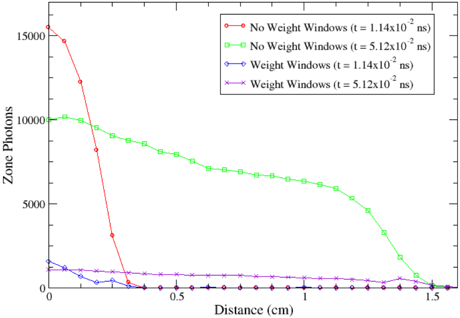
Video



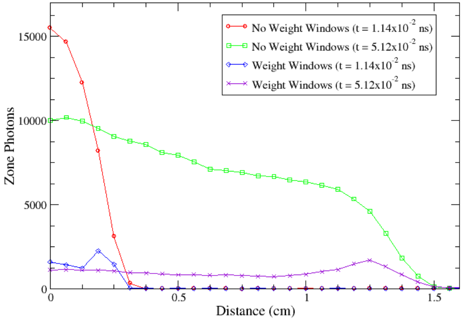
2-D Crooked Pipe Material Temperature Lineout



Particle Flux Using a Split/Roulette Ratio of 5

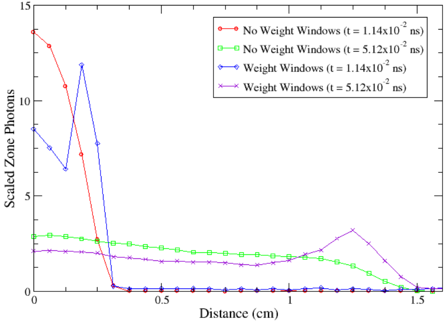


Particle Flux Using a Split/Roulette Ratio of 5000



- In effect, the much larger ratio means there is much more splitting and less roulette

Particle Flux Normalized by the Average Particle Flux



- A value of one means a zone has the average number of photons entering

Simulation Elapsed Times

Table: Simulation Run Times

Simulation Type	Elapsed Time
Converged Solution ($3.5x10^6$)	00:08:38
Without Weight Windows	00:00:58
Width Factor = 0.2	00:11:56
Width Factor = 0.0002	00:16:00

Crooked Pipe Solution Observations

- The solutions obtained without weight windows for the two different time steps do not illustrate complete uniformity with the particle flux
- As expected the particle flux is larger towards the left plane of the simulation, where many particles are originating from the source
- The solution obtained using the lagged weight windows technique exhibits spikes in the number of photons entering into each zone towards the wave front
- This is undesirable because in this problem one would expect to see a uniform particle flux if the proper weights were used

Conclusion

- Using the lagged value of the scalar flux appears to be an inexpensive way of setting the weight window centers for implicit Monte Carlo
- The efficacy of this method was demonstrated on a Marshak wave problem
- This technique is less effective in the optically thin regions of problems as predicted
- We are currently working on an implementation of this method where the weight windows are turned off in thin regions of the problem to avoid the "spiking" of particle densities