ONE-DIMENSIONAL MODELS FOR TRANSPORT IN SOLID CYLINDERS

Ryan G. McClarren
University of Notre Dame

Alex R. Long
Los Alamos National Laboratory

LA-UR-17-29437

THIS WORK IS A BRIDGE BETWEEN TWO OLD PROBLEMS

Admittedly, It Is An Incomplete Bridge

- The so-called Marshak wave problem is a classic problem in high-energy density radiative transfer.
    - Several variations of the problem exist, but the main theme is that a cold medium is subject to a source of radiation and a wave propagates through the medium.
    - The problem usually imagines a slab geometry configuration.
- One-dimensional models for particle transport in evacuated ducts has been used to model the transport of particles using a simplified model.
    - The transport in the duct is modeled using an effective scattering from the duct walls.
- There is active research in developing models for Marshak waves in configurations reminiscent of the duct problem.

THE MARSHAK WAVE PROBLEM

The Classical Work

- The Marshak wave problem has a cold slab with an heat source applied at a particular point.
- Via radiative transfer the medium is heated.
- If the material is optically thick, as the original works assume, the governing equations is a nonlinear diffusion equation.

$$\frac{\partial e(T)}{\partial t} = \frac{\partial}{\partial x}\frac{1}{3\kappa(T)}\frac{\partial}{\partial x}acT^4$$

- Petschek demonstrated how to find similarity solutions.
- Larsen and Pomraning showed that this is the asymptotic limit of a non-equilibrium system.

THE PHYSICS OF FLUIDS VOLUME 1, NUMBER 1 JANUARY-FEBRUARY, 1958

Effect of Radiation on Shock Wave Behavior*

R. E. Marshak
The University of Rochester, Rochester, New York
(Received August 21, 1957)

The fundamental conservation equations governing fluid dynamics and including radiation are written down. The Rankine-Hugoniot conditions are derived for shock waves subjected to radiation flux. Similarity solutions are obtained for the constant density and constant pressure cases. Some results are stated for the combined radiation-fluid dynamics corresponding to a power law dependence of the temperature on the time.

THE NON-EQUILIBRIUM MARSHAK WAVE PROBLEM

G. C. Pomraning
School of Engineering and Applied Science, University of California, Los Angeles, Los Angeles, CA 90024, U.S.A.

(*Received 11 August 1978*)

LOS ALAMOS SCIENTIFIC LABORATORY
OF THE UNIVERSITY OF CALIFORNIA LOS ALAMOS NEW MEXICO

REPORT WRITTEN: May 1960
REPORT DISTRIBUTED: July 29, 1960

THE PENETRATION OF RADIATION
WITH CONSTANT DRIVING TEMPERATURE

Work Done By:
Albert G. Petschek
Ralph E. Williamson
John K. Wooten, Jr.

Report Written By:
Albert G. Petschek
Ralph E. Williamson

SIAM J. APPL. MATH.
Vol. 39, No. 2, October 1980

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0036–139

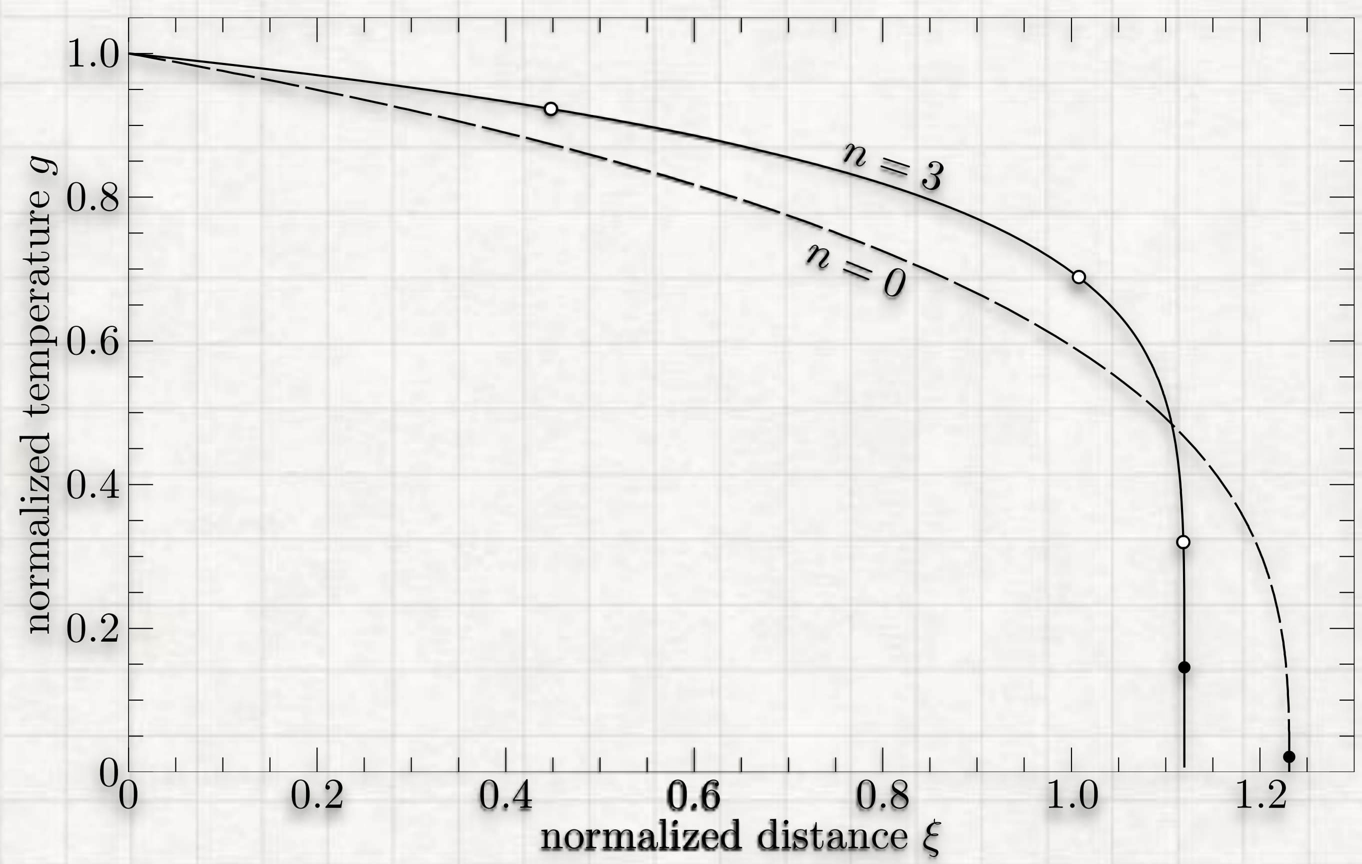
ASYMPTOTIC ANALYSIS OF NONLINEAR MARSHAK WAVES*

E. W. Larsen† and G. C. Pomraning‡

SIMILARITY SOLUTION

Power Law Opacities

- If the opacity has the form
$$\kappa(T) = \kappa_0 T^{-n}$$
- There exists a similarity solution $T(\xi)$ where
$$\xi = C(n)\frac{x}{\sqrt{t}}$$
- The steepness of the wavefront is controlled by the power in the opacity.



The similarity solution for a power-law opacity, from Nelson and Reynolds, LA-UR-09-04551.

EXPERIMENTS DO NOT HAVE SLABS WITH A CONSTANT DRIVING TEMPERATURE

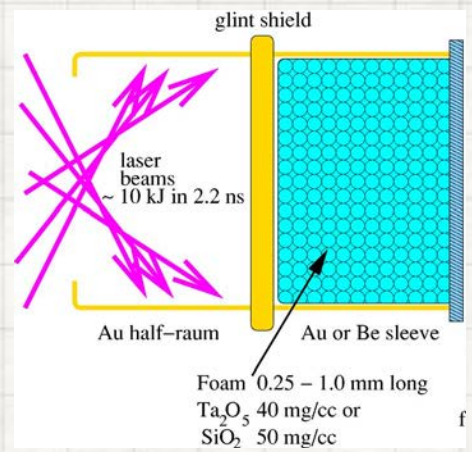
Drive Conditions And Radial Losses Change The Picture

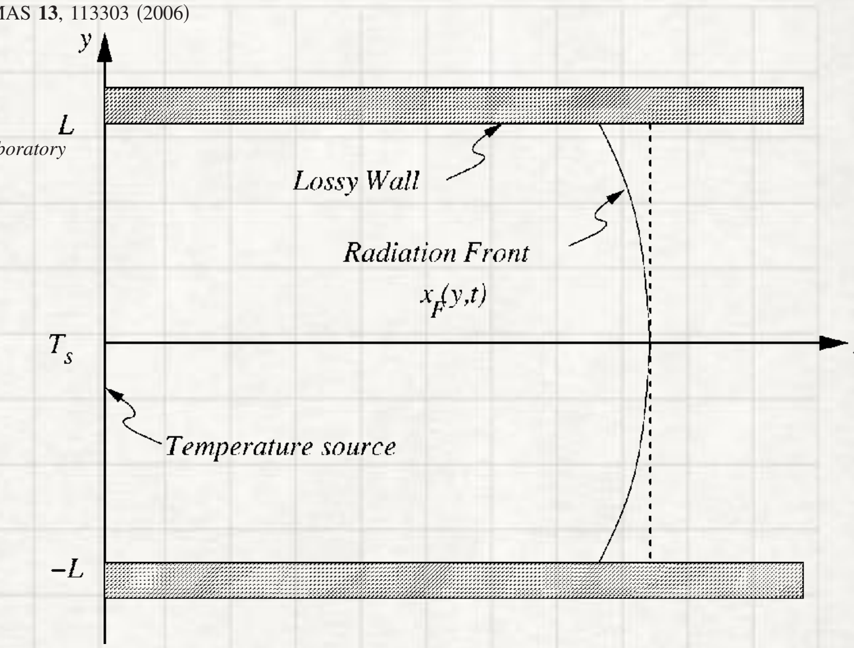
- Hammer and Rosen extended the theory to include a non-constant driving temperature.
- Hurricane and Hammer tried to account for radial losses in the model.
  - Use observation that away from wave front the material temperature changes slowly.
- Other work added radiation energy density terms and material motion into the model.

PHYSICS OF PLASMAS **13**, 113303 (2006)

Bent Marshak waves

O. A. Hurricane[a] and J. H. Hammer
University of California, Lawrence Livermore National Laboratory
P.O. Box 808, Livermore, California 94551





New self-similar radiation-hydrodynamics solutions in the high-energy density, equilibrium diffusion limit

Taylor K Lane and Ryan G McClarren
Department of Nuclear Engineering, Texas A&M University, College Station, TX, USA
E-mail: taylor.lane@neo.tamu.edu and rgm@tamu.edu

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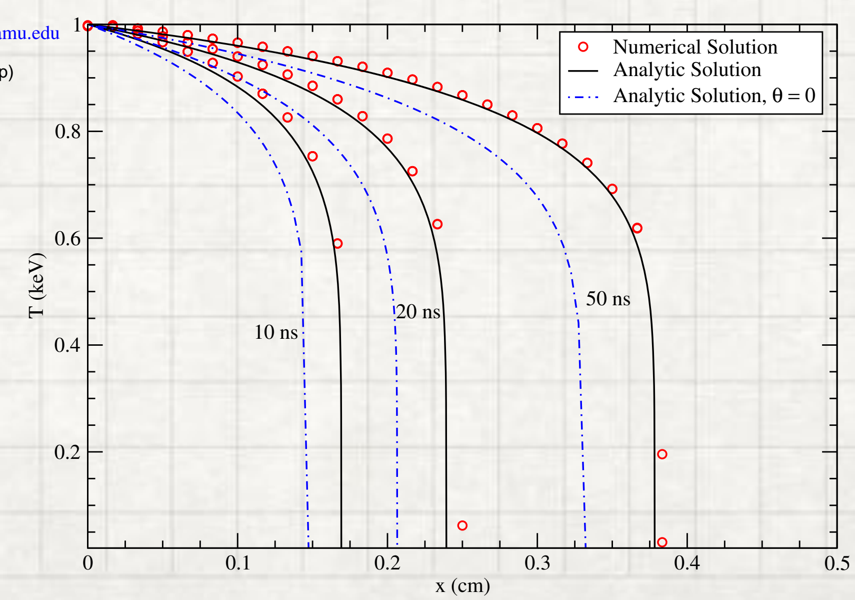


DIFFUSION MODELS OF RADIATIVE TRANSFER IN FINITE SYSTEMS

- The Hurricane-Hammer model points out that behind the wave-front the time dependence of the solution is negligible.
    - This makes the problem here an 2-D Laplace equation.

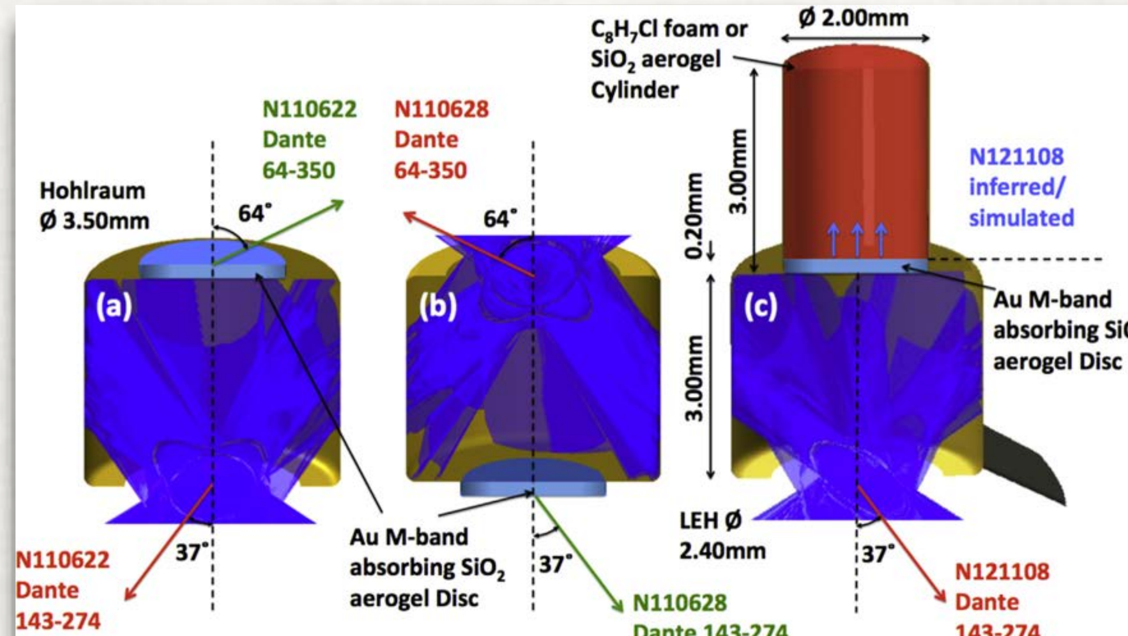
$$\nabla^2 T^4 = 0$$

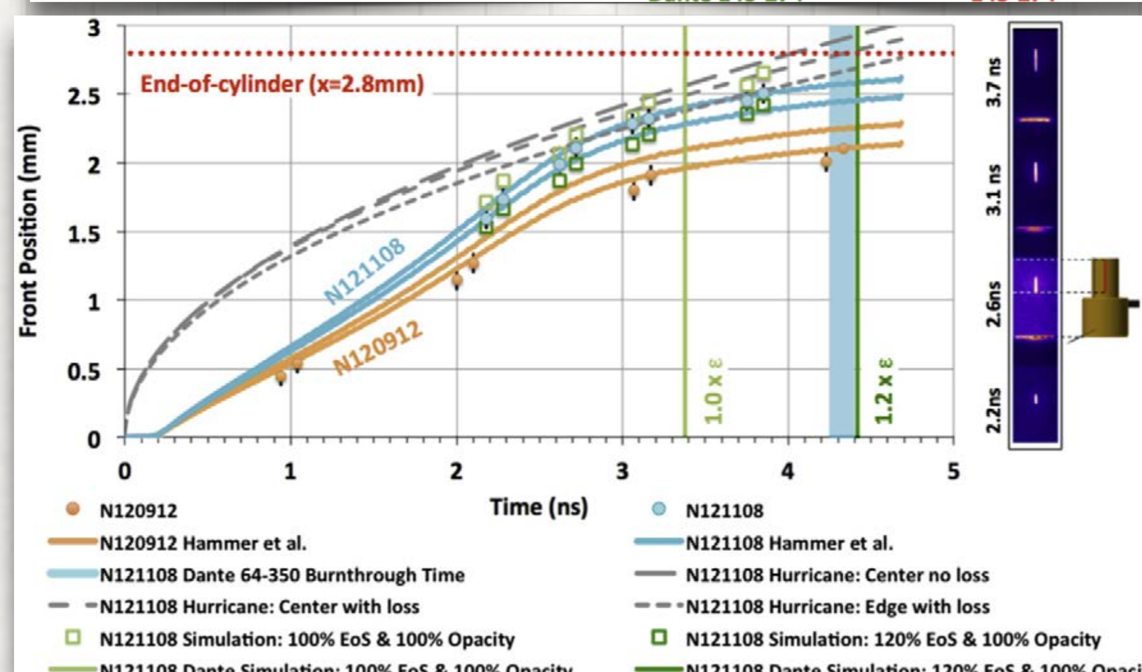
    - In 2-D Cartesian geometry, with x being the direction that the wave is propagating, the wavefront will then be a linear combination of cosines, that determine the curvature of the wavefront.
- Detailed analysis then gives a form for the curvature as a function of boundary conditions, etc., and an equation for the propagation of the wavefront.
- Hammer and Rosen also use the slow change behind the wavefront, but they use this to derive the results from changing boundary temperatures for 1-D problems, and solve problems regarding supersonic and subsonic diffusion waves.

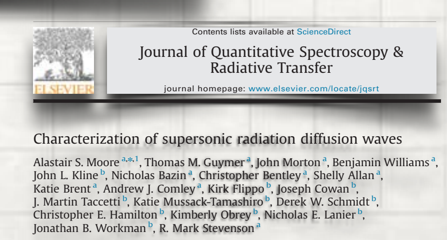
COMPARISONS WITH EXPERIMENT SHOW THESE MODELS ARE REASONABLE

Improvement Is Possible

- A recent paper by Moore, et al. compares Marshak waves created at NIF with the different models of Hammer and Hurricane.
- There seems to be a gap between analytic theory and full-blown radiation hydrodynamics simulations.
- Is a simple, ordered, approximation to the radiative transfer in the experiment possible?






Contents lists available at ScienceDirect

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Characterization of supersonic radiation diffusion waves

Alastair S. Moore[a,*,1], Thomas M. Guymer[a], John Morton[a], Benjamin Williams[a], John L. Kline[b], Nicholas Bazin[a], Christopher Bentley[a], Shelly Allan[a], Katie Brent[a], Andrew J. Comley[a], Kirk Flippo[b], Joseph Cowan[b], J. Martin Taccetti[b], Katie Mussack-Tamashiro[b], Derek W. Schmidt[b], Christopher E. Hamilton[b], Kimberly Obrey[b], Nicholas E. Lanier[b], Jonathan B. Workman[b], R. Mark Stevenson[a]

[a] AWE Aldermaston, Reading RG7 4PR, United Kingdom
[b] Los Alamos National Laboratory, Los Alamos, NM 87545 USA

MEANWHILE, 1-D MODELS WERE BEING DEVELOPED FOR TRANSPORT IN A DUCT

Sometimes By The Same People

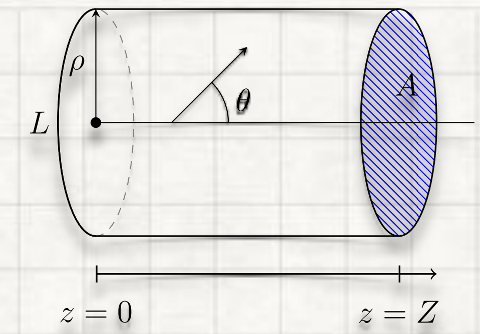
- Consider particles traveling in a duct of material that is surrounded by a wall that can reflect a fraction, c, of the particles back into the duct.
- Most of the models were developed for evacuated ducts where particles enter at one end.
- In an evacuated duct, the transport equation is written as

$$\mu\frac{\partial\psi}{\partial z} + \sqrt{1-\mu^2}\left(\cos\gamma\frac{\partial\psi}{\partial x} + \sin\gamma\frac{\partial\psi}{\partial y}\right) = 0$$

- Using a Galerkin procedure, a simplified 1-D model can be derived

$$\mu\frac{\partial\Psi}{\partial z} + \sqrt{1-\mu^2}\mathbf{A}\Psi = \frac{2c}{\pi}\mathbf{B}\int_{-1}^{1}\sqrt{1-\mu'^2}\Psi(z,\mu')\,d\mu'$$

$$\psi(x,y,z,\mu,\gamma) = \sum_{n=1}^{N} b_n(x,y,\gamma)\Psi_n$$



NUCLEAR SCIENCE AND ENGINEERING: 93, 13-30 (1986)

One-Dimensional Models for Neutral Particle Transport in Ducts

Edward W. Larsen
Los Alamos National Laboratory, Los Alamos, New Mexico 87545
and
Fausto Malvagi and G. C. Pomraning
University of California at Los Angeles, School of Engineering and Applied Science Los Angeles, California 90024

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TRANSPORT THEORY AND STATISTICAL PHYSICS, 13(5), 599-614 (1984)

A ONE-DIMENSIONAL MODEL FOR THREE-DIMENSIONAL TRANSPORT IN A PIPE

Edward W. Larsen
University of California
Los Alamos National Laboratory
Los Alamos, NM 87544

TRANSPORT THEORY AND STATISTICAL PHYSICS, 13(5), 567-598 (1984)

A STATISTICAL MODEL OF TRANSPORT IN A VACUUM
by
A.K. Prinja and G.C. Pomraning
School of Engineering and Applied Science
University of California at Los Angeles
Los Angeles, California 90024

A singular integral equation formulation and solution for transport in semi-infinite ducts

Fausto Malvagi and G. C. Pomraning
School of Engineering and Applied Science, University of California, Los Angeles, California 90024

EXTENSIONS TO THIS DUCT WORK

- A quadratic expansion was introduced by Garcia, et al. (3 basis functions).
- Multigroup models for the transport also exist.
- Prinja modified the model to allow particles to re-enter the duct at different places (non-local reflection).
- Many papers on the efficient solution of the models.
- Models used in shielding and acoustics.

ON THE SOLUTION OF A NONLOCAL TRANSPORT EQUATION BY THE WIENER-HOPF METHOD

ANIL K. PRINJA

Chemical and Nuclear Engineering Department
The University of New Mexico
Albuquerque, NM 87131, USA

Dedicated to Prof. M.M.R. Williams on the occasion of his 60th birthday

One-dimensional transport equation models for sound energy propagation in long spaces: Simulations and experiments

Yun Jing[a] and Ning Xiang
Graduate Program in Architectural Acoustics, School of Architecture, Rensselaer Polytechnic Institute, Troy, New York 12180

(Received 8 September 2009; revised 5 January 2010; accepted 7 January 2010)

The Third Basis Function Relevant to an Approximate Model of Neutral Particle Transport in Ducts

Roberto D. M. Garcia*
Centro Técnico Aeroespacial, Instituto de Estudos Avançados
12231-970 São José dos Campos, SP, Brazil and HSH Scientific Computing
Rua Carlos de Campos, 286, 12242-540 São José dos Campos, SP, Brazil

and

Shizuca Ono and Wilson J. Vieira
Centro Técnico Aeroespacial, Instituto de Estudos Avançados
12231-970 São José dos Campos, SP, Brazil

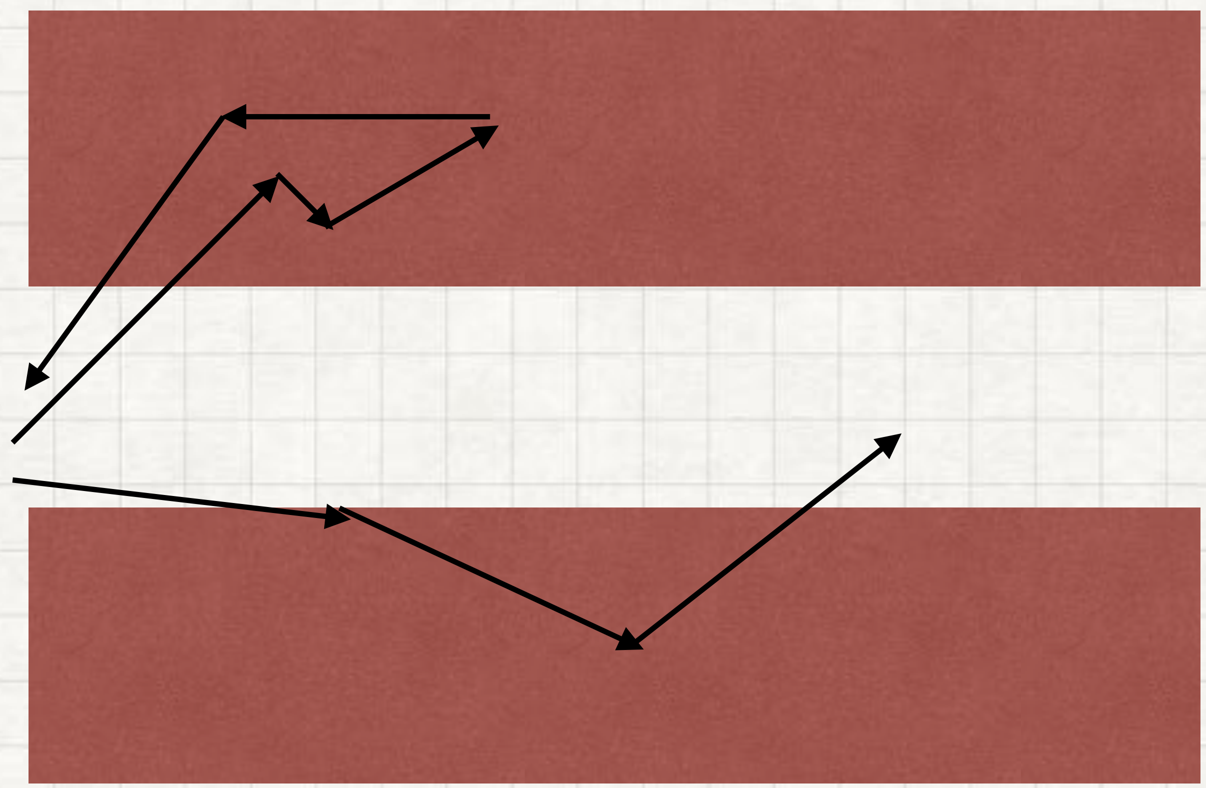
Approximate One-Dimensional Models for Monoenergetic Neutral Particle Transport in Ducts with Wall Migration

Arnulfo Gonzalez and Ryan G. McClarren
Department of Nuclear Engineering Texas A&M University College Station, TX, USA

WALL MIGRATION OF NEUTRONS

Neutrons collisions don't take place at the wall, rather inside the wall.

As a result, particles can re-enter duct ahead of or behind where the particle left the duct.



Wall-Scattering now has a non-local kernel

$$\mathbf{B} \int_0^Z dz' \int_{-1}^{1} (1-\mu'^2)^{\frac{1}{2}}\, K(z' \to z)\, \vec{\psi}(z,\mu')\, d\mu'.$$

Example kernel function

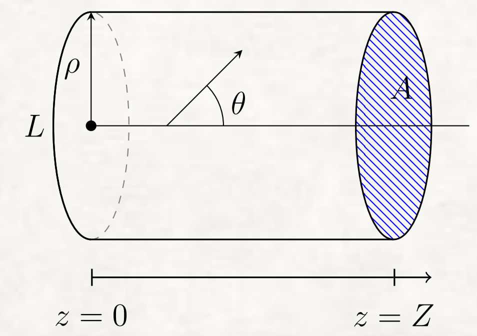
$$\int_0^Z K(z' \to z)\, dz' = \int_0^Z \frac{\lambda}{2} \exp(-\lambda |z - z'|)\, dz'$$

THE CYLINDER IN THE MARSHAK WAVE PROBLEM IS A DUCT

It Is Just Not An Evacuated Duct

- The radiative transfer in a cylinder is, in some sense, the opposite of the duct model.
  - Collisions take place in the cylinder and the walls have a reflection probably that could be zero.
- The 1-D duct models make no assumptions that the walls must scatter particles back into the domain.
- Furthermore, as pointed out early on in their history, there is no limitation to extending to non-evacuated ducts in the models.
  - They are just typically applied to evacuated regions.
- The curvature in the wave-front, time-dependence of drive, etc. could be included in the model.
- This would be a simplified model that captures the effects the 1-D diffusion models lack.

WE DEVELOP A MODEL FOR TIME-DEPENDENT TRANSPORT IN A SOLID CYLINDER



- We consider a linear transport problem:

$$\frac{1}{v}\frac{\partial \Psi}{\partial t} + (1-\mu^2)^{\frac{1}{2}}\left(\cos\varphi\frac{\partial}{\partial x} + \sin\varphi\frac{\partial}{\partial y}\right)\Psi + \mu\frac{\partial \Psi}{\partial z} + \sigma_{\mathrm{t}}\Psi = \frac{\sigma_{\mathrm{s}}}{4\pi}\int\limits_{-1}^{1} d\mu' \int\limits_{0}^{2\pi} d\varphi'\, \Psi(x,y,z,\mu',\varphi',t) + \frac{Q}{4\pi}$$

$$\Psi(x,y,0,\mu,\varphi,t) = g_l(x,y,\mu,\varphi,t), \qquad h(x,y) < 0, \quad \mu > 0,$$

$$\Psi(x,y,Z,\mu,\varphi,t) = g_r(x,y,\mu,\varphi,t), \qquad h(x,y) < 0, \quad \mu < 0.$$

$$\Psi(x,y,z,\mu,\varphi,t) = 0, \qquad h(x,y) = 0, \quad \omega\cdot\mathbf{n} < 0,$$

$$\boldsymbol{\omega} = (\cos\varphi, \sin\varphi, 0)$$

- We then propose an expansion in basis functions that contain the variation in x,y, and the azimuthal angle.

$$\Psi(x,y,z,\mu,\varphi) \approx \sum_{i=1}^{I} \psi_i(z,\mu,t) b_i(x,y,\varphi).$$

Here $I \leq 3$. The basis functions are

$$b_1(x,y,\varphi) = 1,$$

$$b_2(x,y,\varphi) = u[D(x,y,\varphi) - v],$$

$$b_3(x,y,\varphi) = r[D(x,y,\varphi) - v][D(x,y,\varphi) - v - q] - r/u^2.$$

THE GALERKIN PROJECTION TO A 1-D MODEL

- The function D gives the distance from a point to the edge of the cylinder along the direction **ω**. For a cylinder this is

$$D(x,y,\varphi) = \mathbf{r}\cdot\omega + [(\mathbf{r}\cdot\omega)^2 + \rho^2 - x^2 - y^2]^{\frac{1}{2}}, \qquad \mathbf{r} = (x,y,0)$$

- The basis expansion is substituted into the transport equation, and the resulting equation is integrated against the basis functions to get

$$\frac{1}{v}\frac{\partial \psi_i}{\partial t} + \mu\frac{\partial \psi_i}{\partial z} + \sigma_{\mathrm{t}}\psi_i + \sqrt{1-\mu^2}\sum_{j=1}^{3} a_{ij}\psi_j(z,\mu,t) = \frac{\sigma_{\mathrm{s}}}{2}\phi_i(z,t) + Q_i,$$

$$Q_i = \frac{1}{2\pi A}\int_R dx\,dy\int_0^{2\pi} d\varphi\, b_i(x,y,\varphi)\frac{Q}{4\pi}.$$

- These equations are a set of coupled 1-D transport equations. The effective total cross-section depends on μ.

- The coupling does make the equations look like a multigroup system.

THE MATRIX ELEMENTS ARE NOT NECESSARILY POSITIVE

From Garcia, Ono, And Veira (2000)

$$a_{11} = b_{11} = L/(\pi A) \ ,$$

$$a_{12} = b_{12} = u - uvL/(\pi A) \ ,$$

$$a_{21} = b_{21} = -uvL/(\pi A) \ ,$$

$$a_{22} = u^2 v^2 L/(\pi A) \ ,$$

$$b_{22} = -uv[u - uvL/(\pi A)] \ .$$

$$a_{13} = b_{13} = -qr + (v^2 + qv - 1/u^2) rL/(\pi A) \ ,$$

$$a_{31} = b_{31} = (v^2 + qv - 1/u^2) rL/(\pi A) \ ,$$

$$a_{23} = (2r/u) - (v^2 + qv - 1/u^2) uvrL/(\pi A) \ ,$$

$$b_{23} = uv[qr - (v^2 + qv - 1/u^2) rL/(\pi A)] \ ,$$

$$a_{32} = -(v^2 + qv - 1/u^2) uvrL/(\pi A) \ ,$$

$$b_{32} = r(v^2 + qv - 1/u^2)[u - uvL/(\pi A)] \ ,$$

$$a_{33} = (v^2 + qv - 1/u^2)^2 r^2 L/(\pi A) \ ,$$

and

$$b_{33} = -r(v^2 + qv - 1/u^2) \times [qr - (v^2 + qv - 1/u^2) rL/(\pi A)] \ .$$

$$q = 8\rho\left[\frac{9\pi}{5}(9\pi^2 - 64)^{-1} - \frac{2}{3\pi}\right]$$

$$u = \left\{\frac{1}{2\pi A}\int_R \int_0^{2\pi} [D(x, y, \boldsymbol{\omega}) - v]^2 d\varphi dx dy\right\}^{-1/2}$$

$$r = \rho^{-2}\left[1 - \frac{576}{25}(9\pi^2 - 64)^{-1}\right]^{-1/2}$$

$$v = \frac{1}{2\pi A}\int_R \int_0^{2\pi} D(x, y, \boldsymbol{\omega})\, d\varphi dx dy \ .$$

WE SOLVE THE EQUATIONS WITH A DISCRETE ORDINATES SCHEME

Off-Diagonal Terms Are Updated Through Source Iteration

- We compute transport sweeps as

$$\mu_k \frac{\partial}{\partial z}\psi_{ik}^{\ell} + (\sigma_{\mathrm{t}}^* + a_{ii})\psi_{ik}^{\ell} = \sum_{i\neq j}^{3} a_{ij}\sqrt{1-\mu_k^2}\psi_{jk}^{\ell-1} + \frac{\sigma_{\mathrm{s}}}{2}\phi_i^{\ell-1} + Q_{ik}^*$$

- k is the angle index, and superscript ℓ is the iteration index. The asterisks denote changes made to write the time-dependent equation in quasi-steady form.

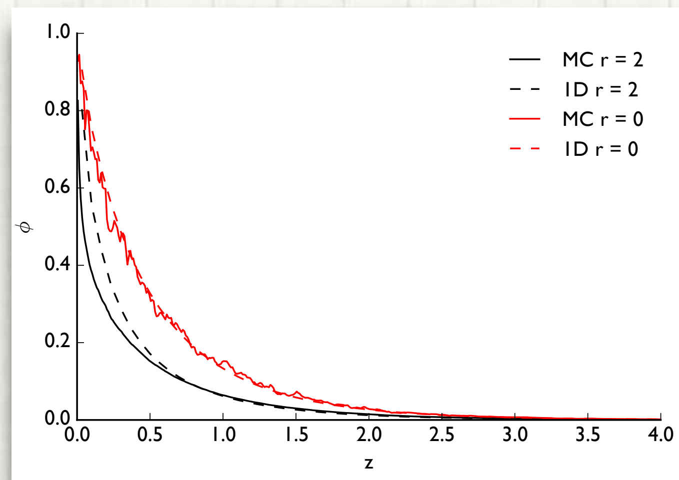
$$\sigma_{\mathrm{t}}^* = \sigma_{\mathrm{t}} + \frac{1}{v\Delta t} \qquad Q_{ik}^* = Q_i + \frac{\psi_{ik}^{\mathrm{old}}}{v\Delta t}$$

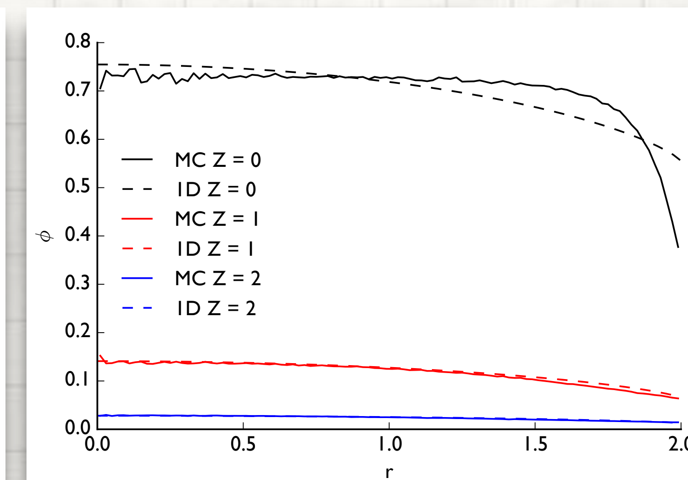
- One could imagine developing acceleration strategies for these equations, but we have not needed them in our work to date.

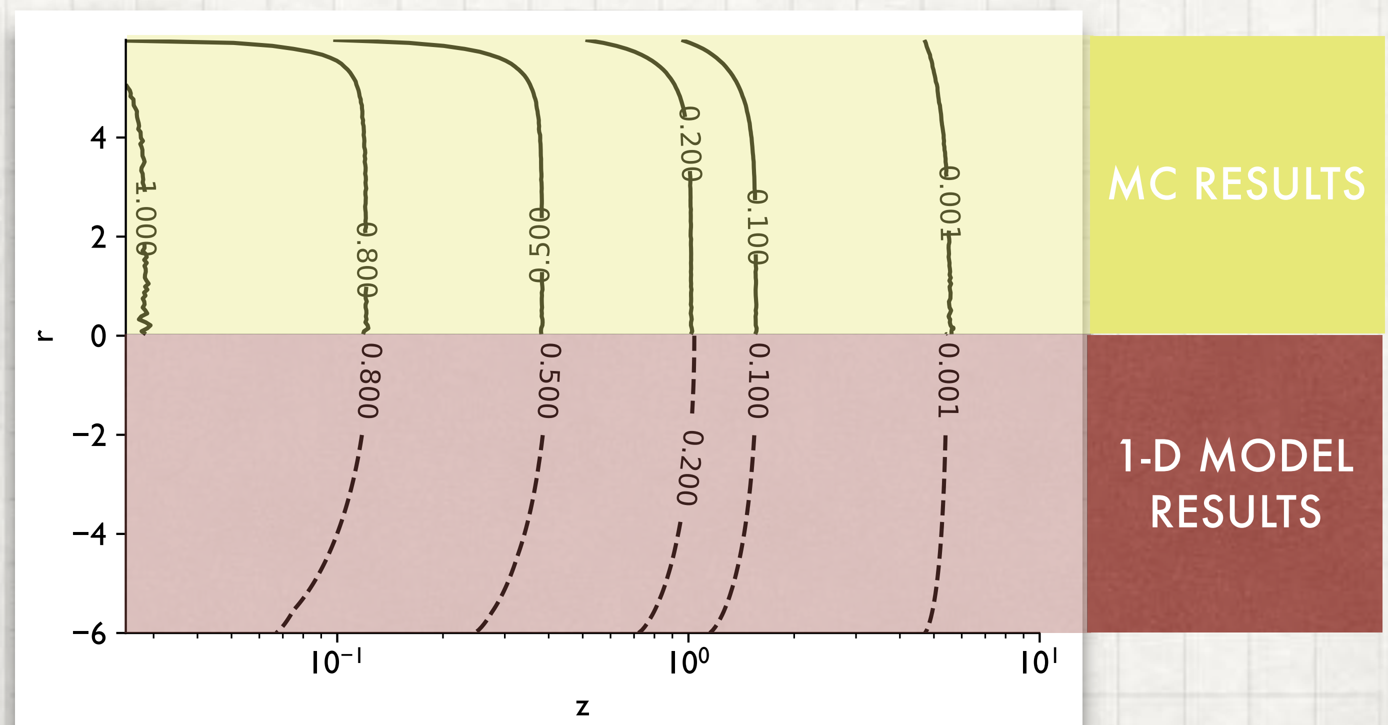
STEADY-STATE RESULTS

Comparison With Monte Carlo

- We solve a problem of a cylinder of a single material with unit total cross-section and length 10.
- We vary the radius between 2 and 6, and the scattering ratio from 0 to 1.
- A unit incident, isotropic flux is imposed at z=0.
- Comparisons are made with calculations from Milagro from LANL.
- Contour maps will have MC results at positive r and 1-D results at “negative” r.

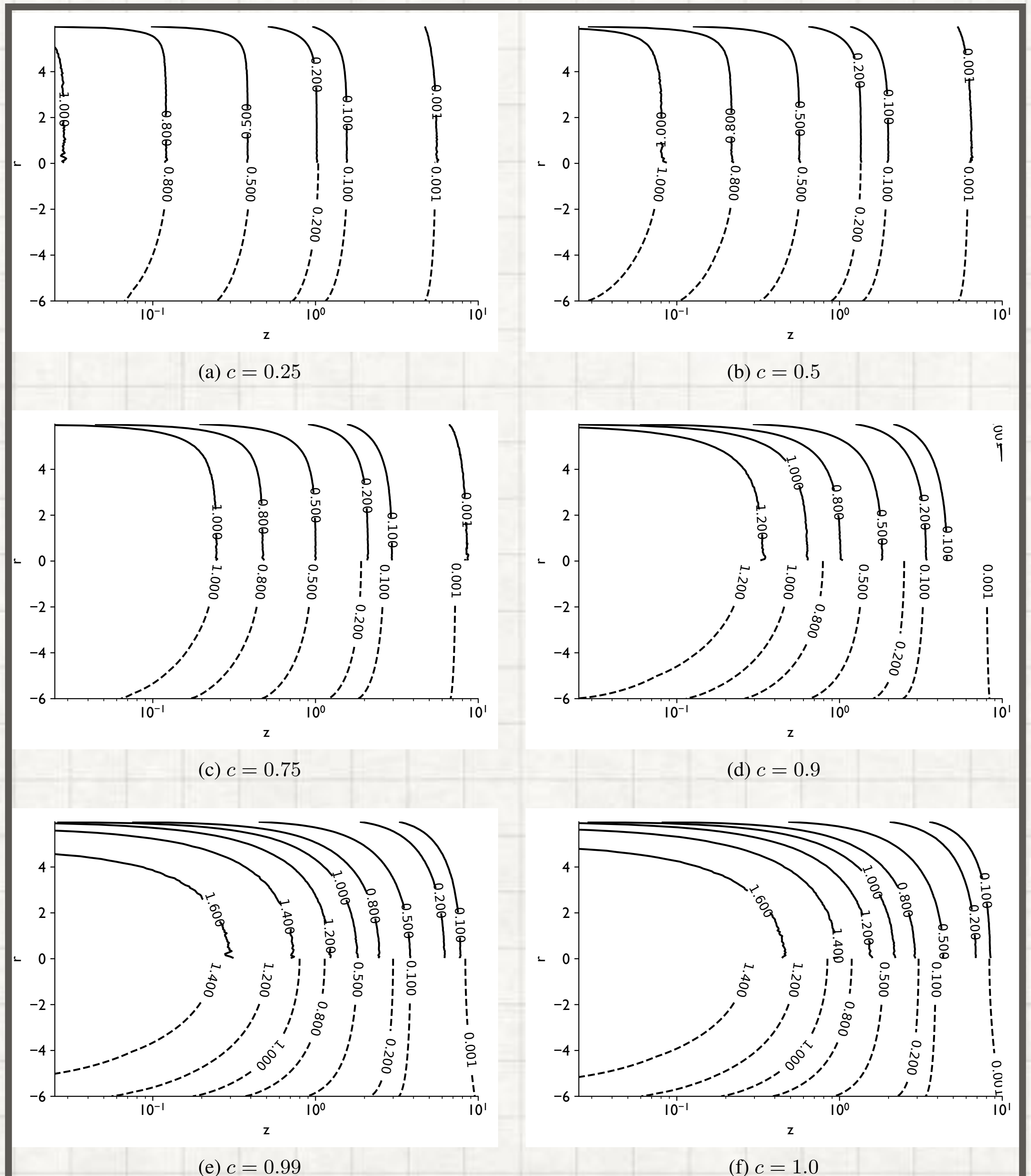






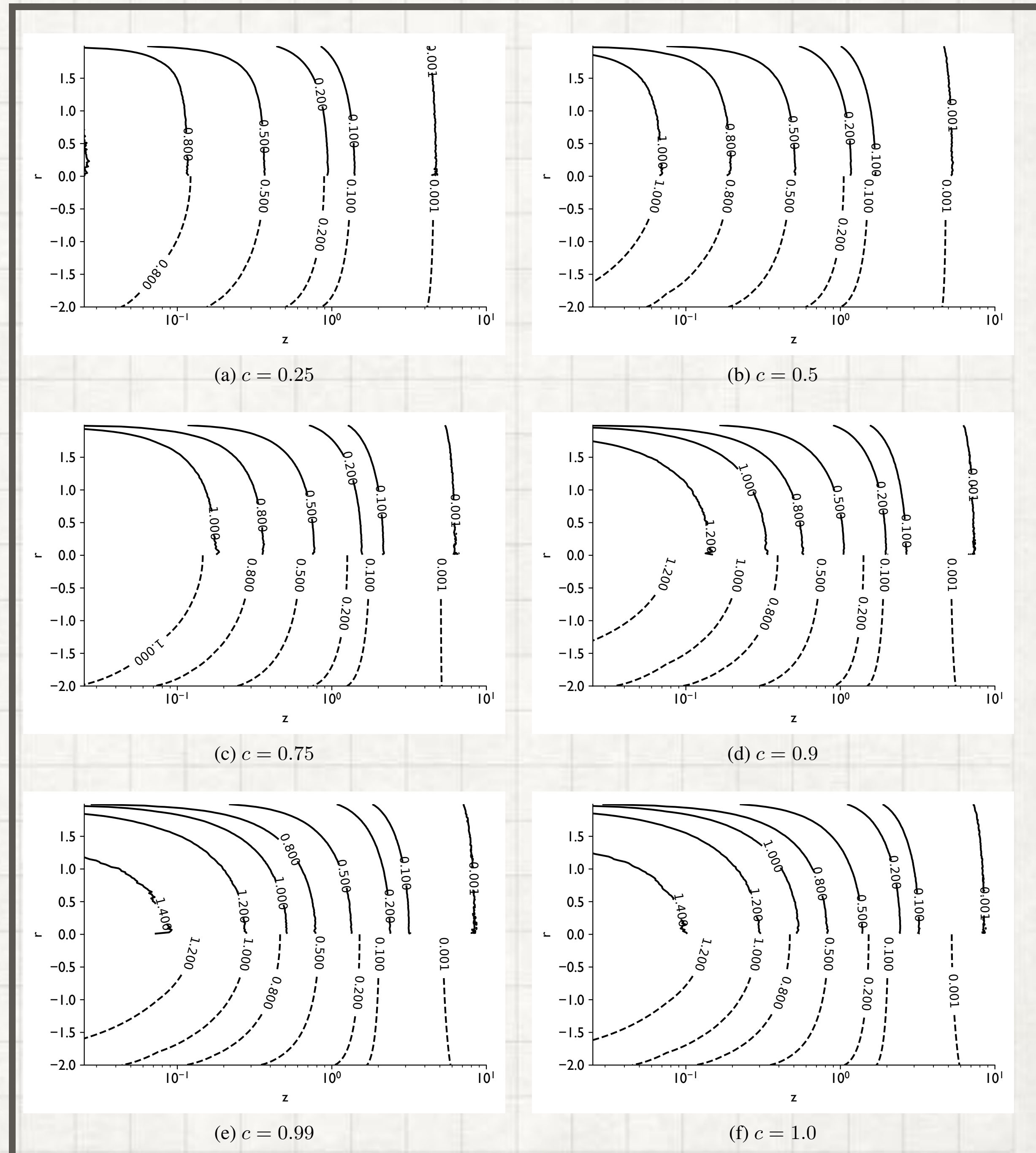
1-D MODELS PERFORM BETTER WITH LOW SCATTERING RATIO

3 Basis Functions



(a) $c = 0.25$ (b) $c = 0.5$ (c) $c = 0.75$ (d) $c = 0.9$ (e) $c = 0.99$ (f) $c = 1.0$

Radius 6

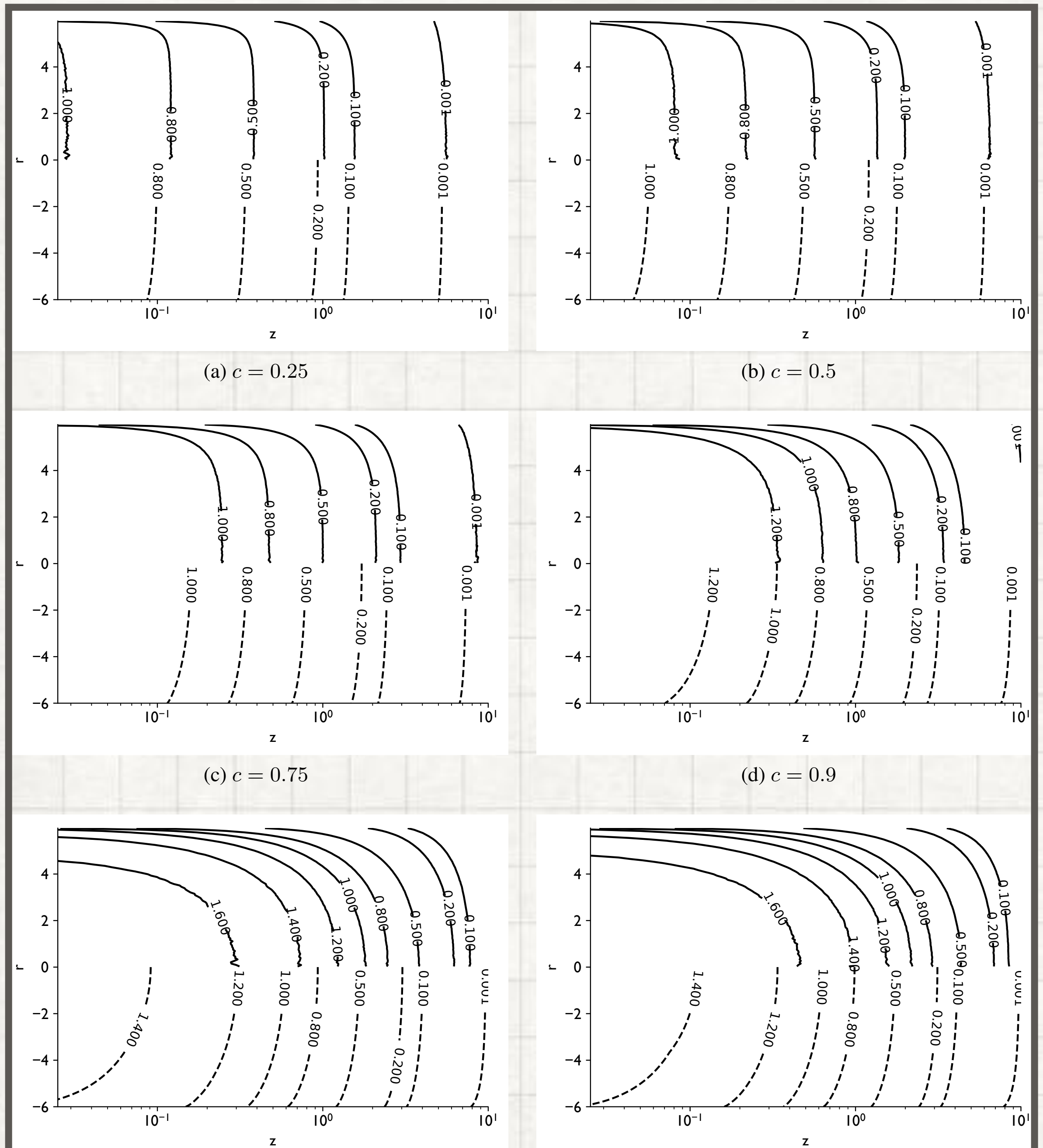


(a) $c = 0.25$ (b) $c = 0.5$ (c) $c = 0.75$ (d) $c = 0.9$ (e) $c = 0.99$ (f) $c = 1.0$

Radius 2

THE QUADRATIC MODEL IS NECESSARY

2 And 1 Basis Functions Are Significantly Worse

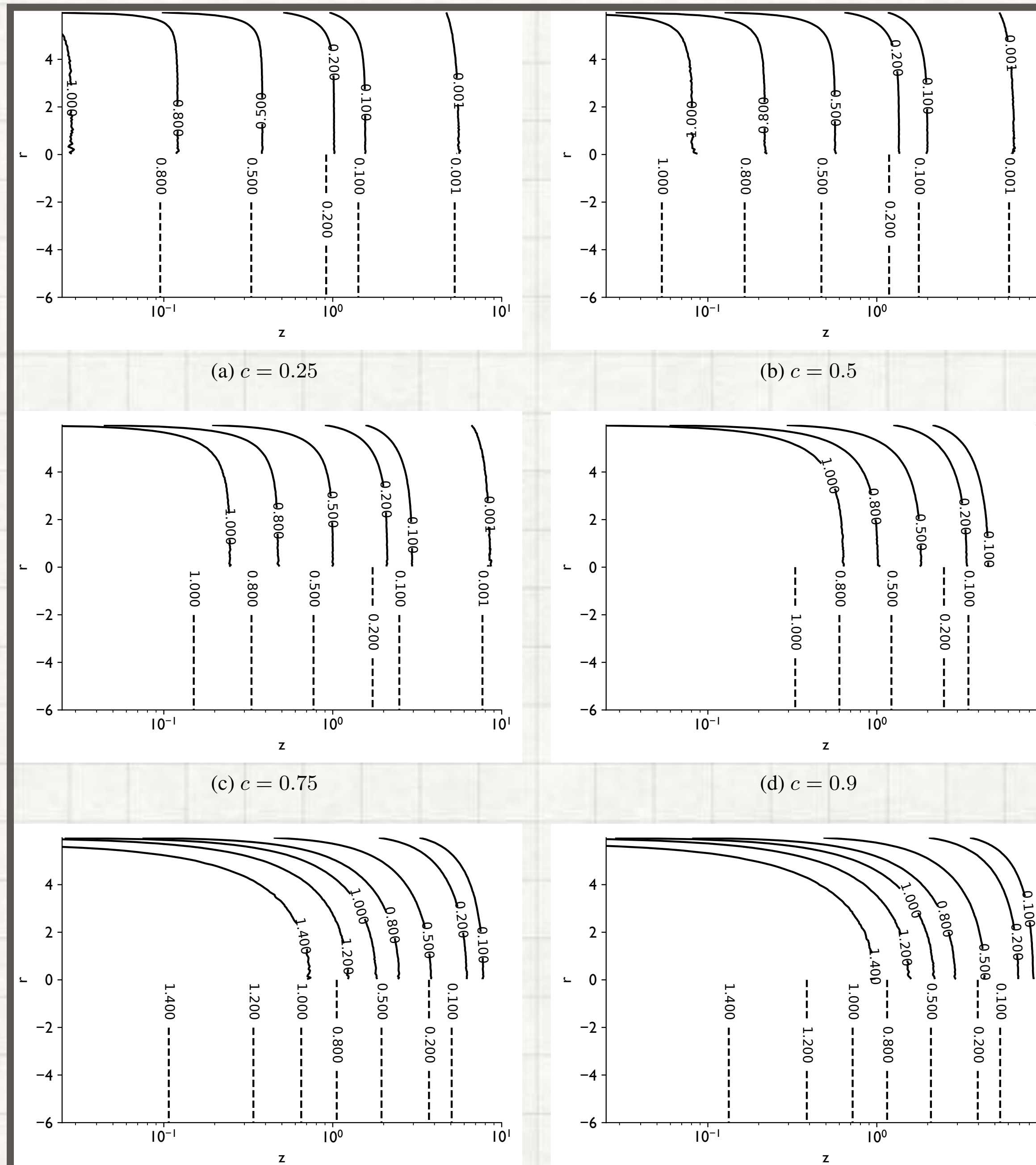


(a) $c = 0.25$ (b) $c = 0.5$

(c) $c = 0.75$ (d) $c = 0.9$

(e) $c = 0.99$ (f) $c = 1.0$

2-Basis Functions, Radius 6



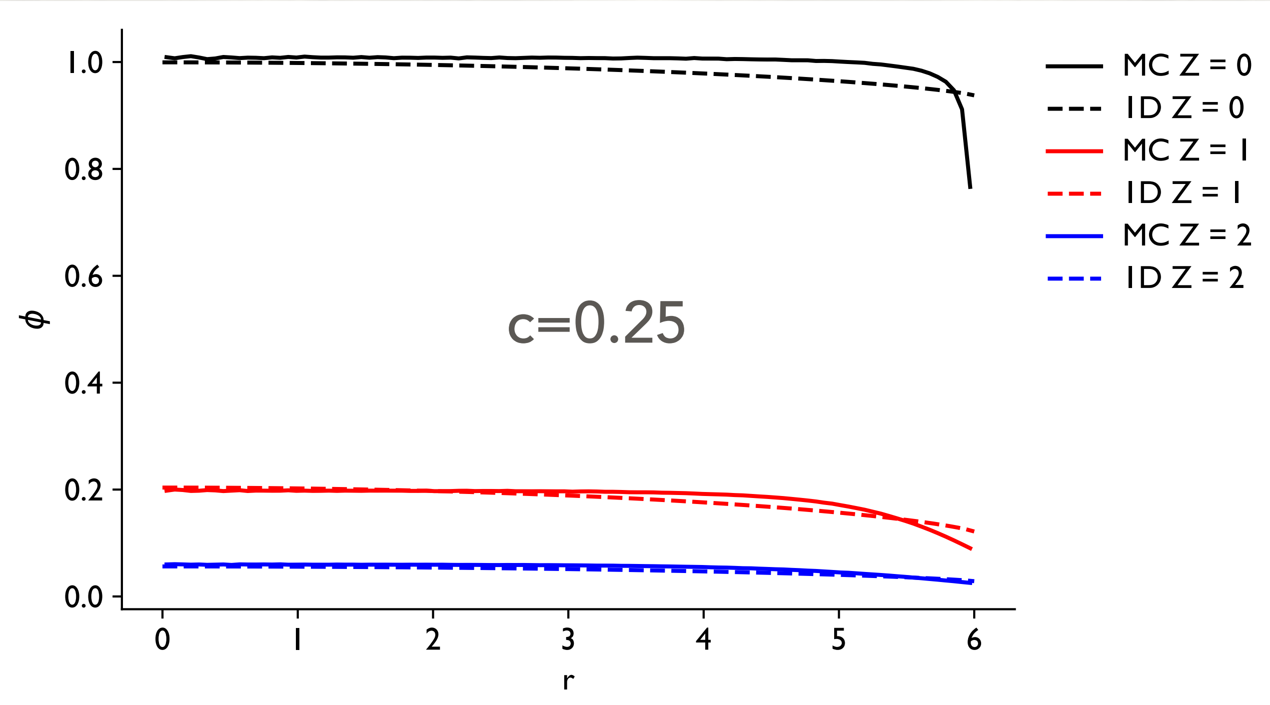
(a) $c = 0.25$ (b) $c = 0.5$

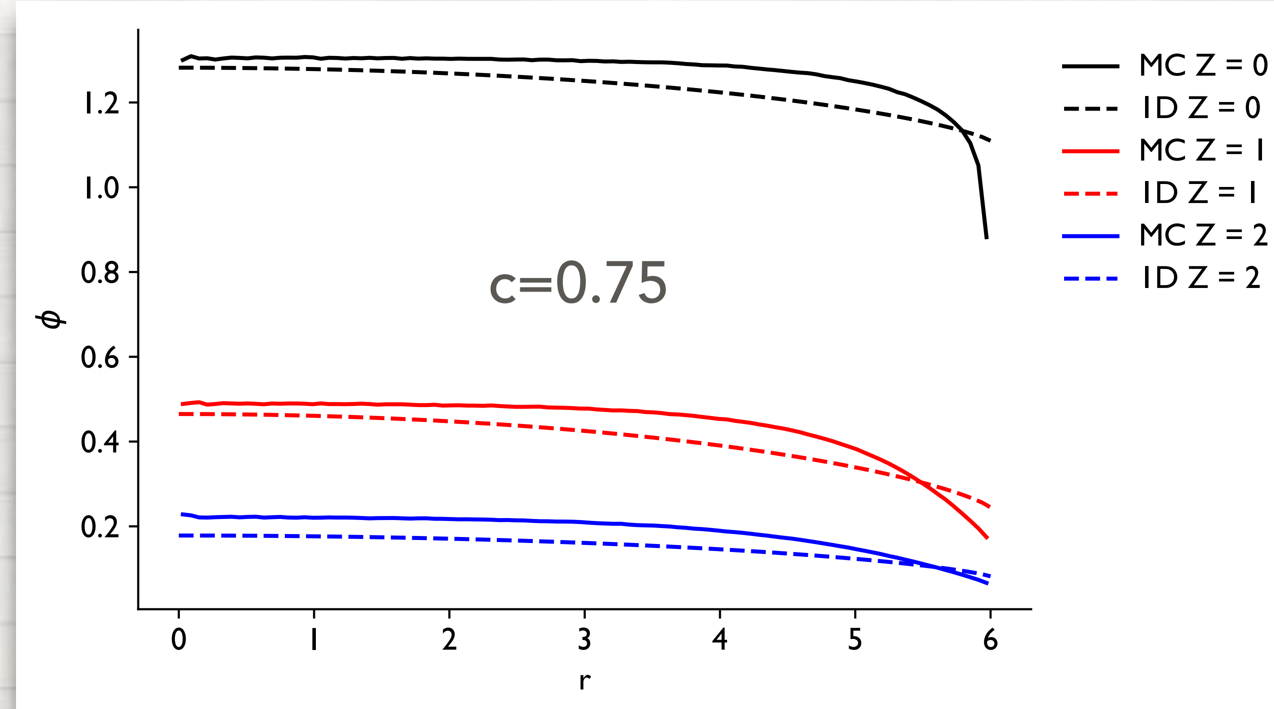
(c) $c = 0.75$ (d) $c = 0.9$

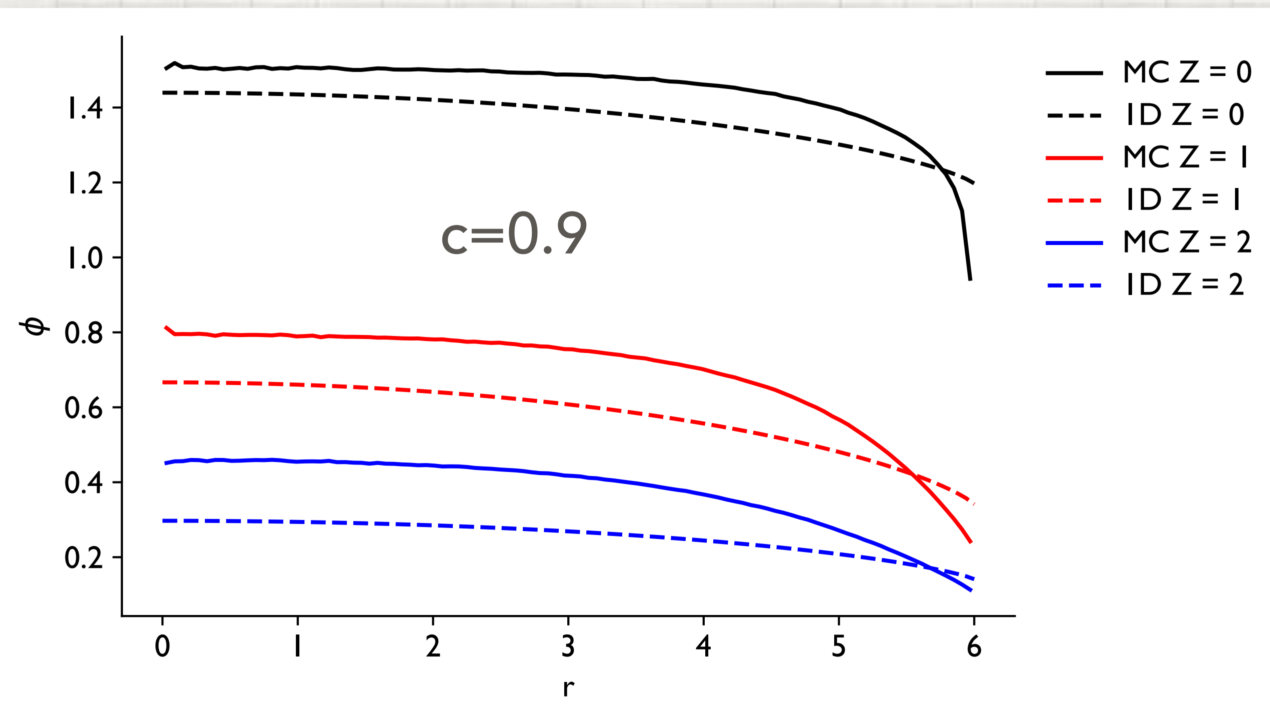
(e) $c = 0.99$ (f) $c = 1.0$

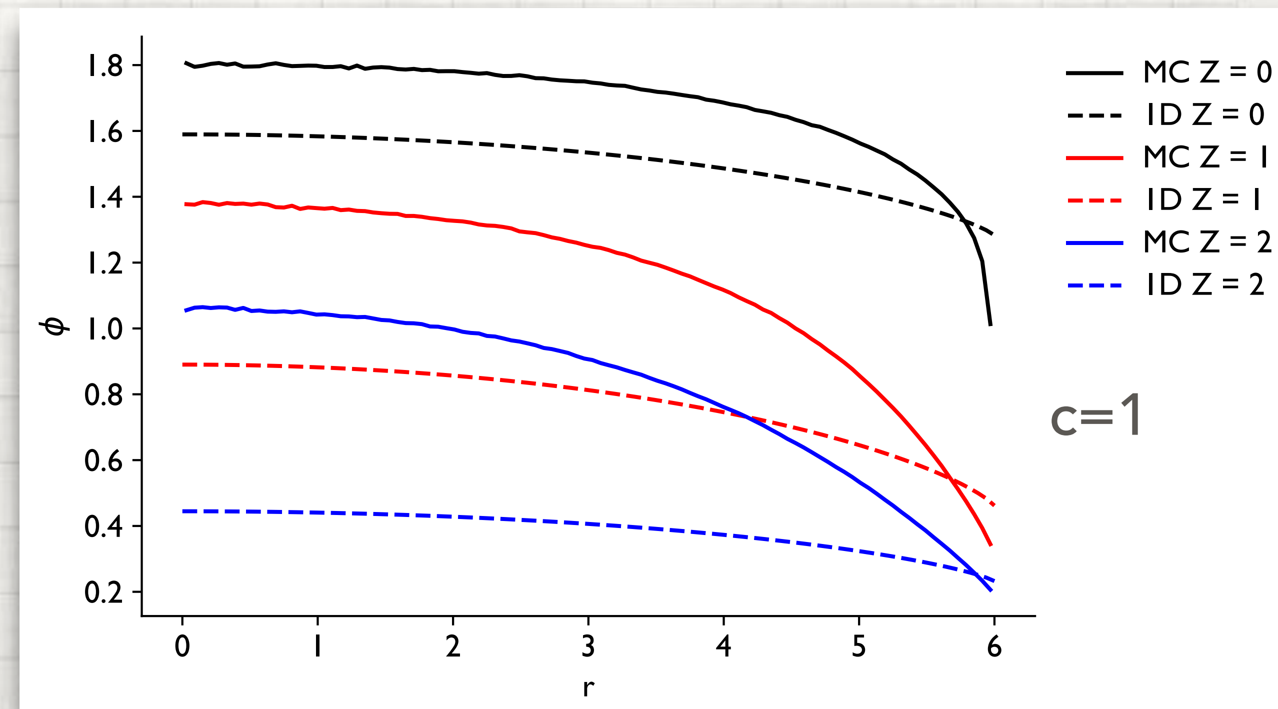
1-Basis Function, Radius 6

3 BASIS FUNCTION SOLUTIONS VERSUS RADIUS



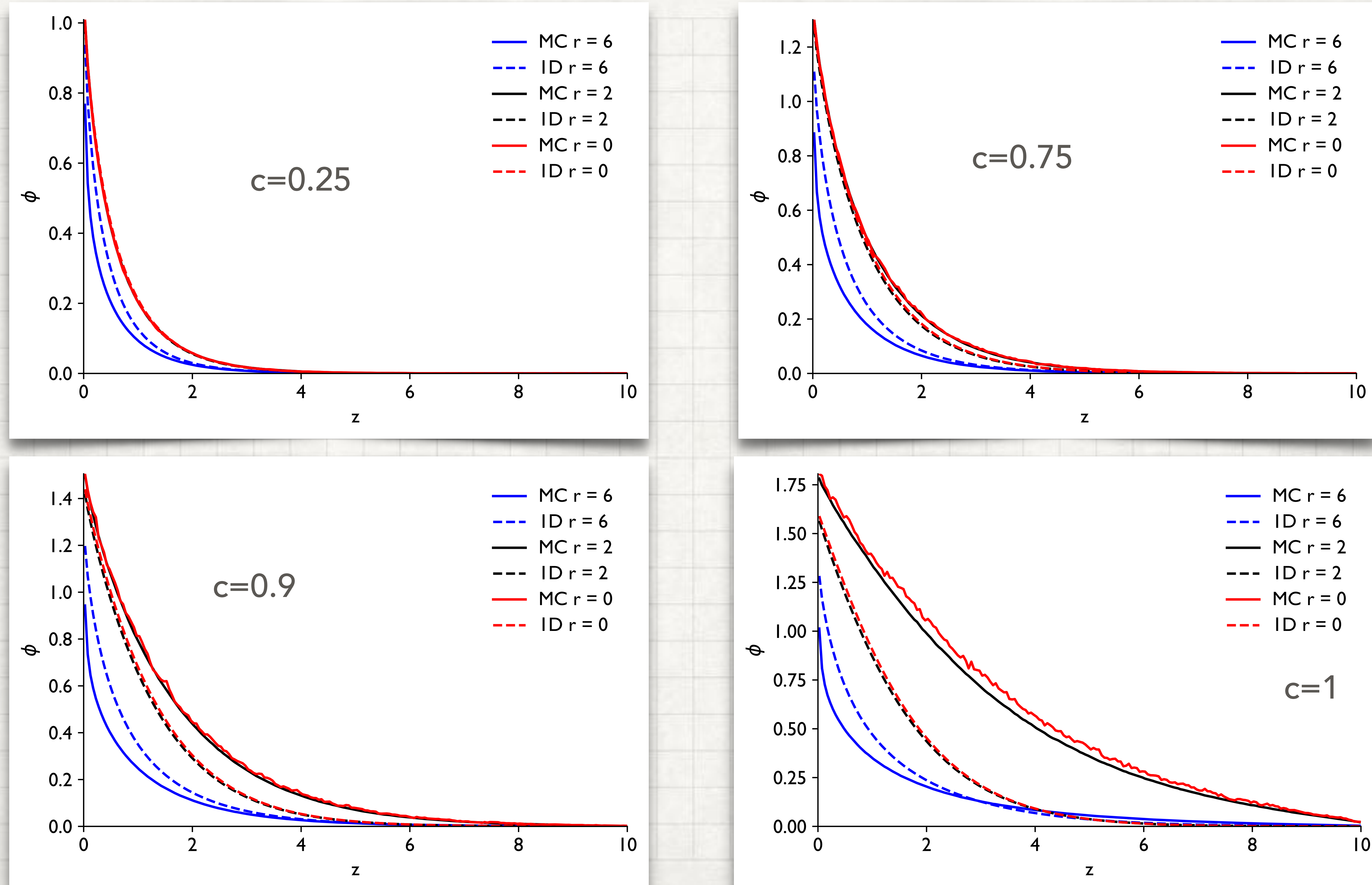






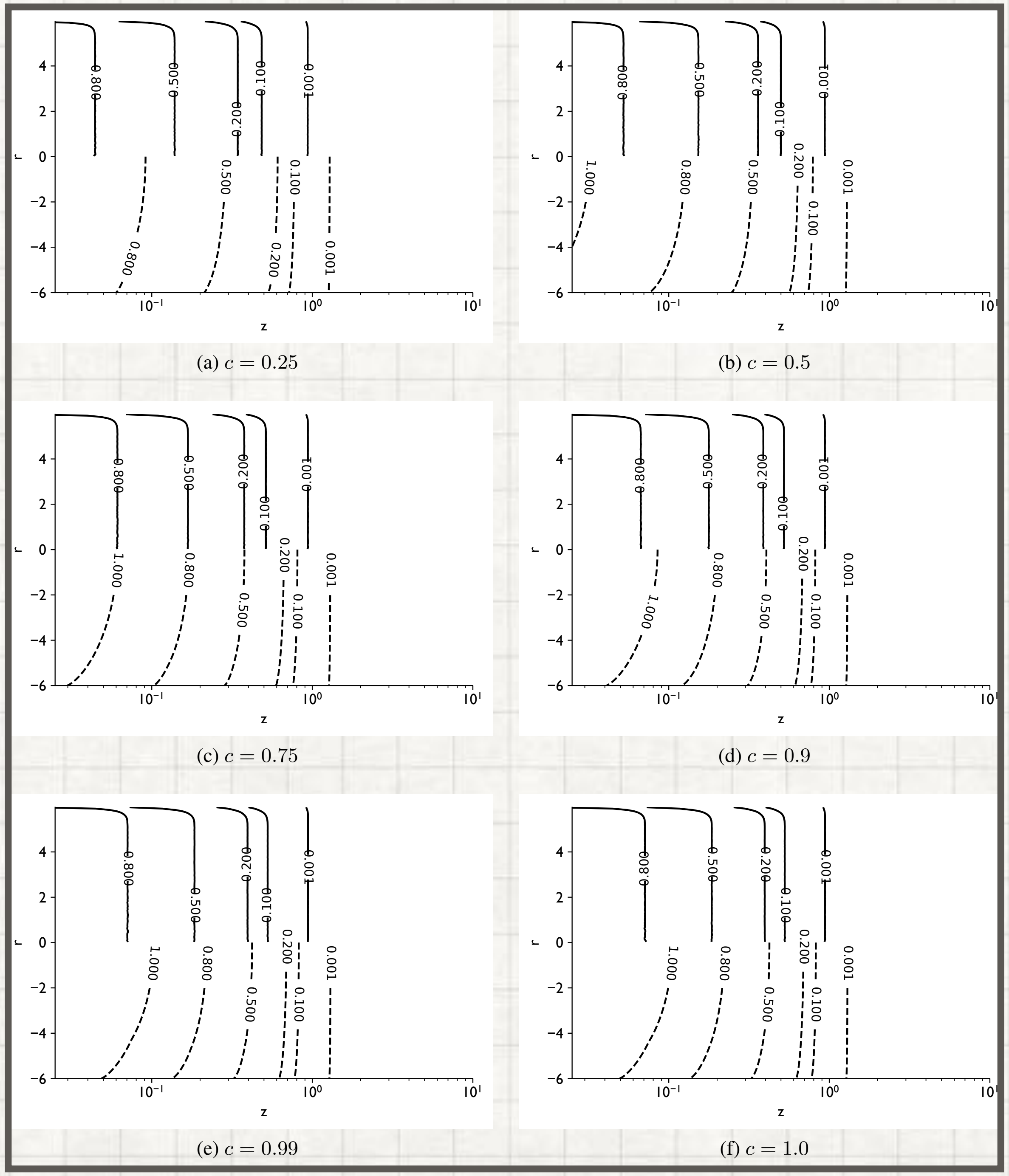
BIGGEST DISCREPANCY IS NEAR RADIAL EDGE

3 Basis Function Solutions, R=6

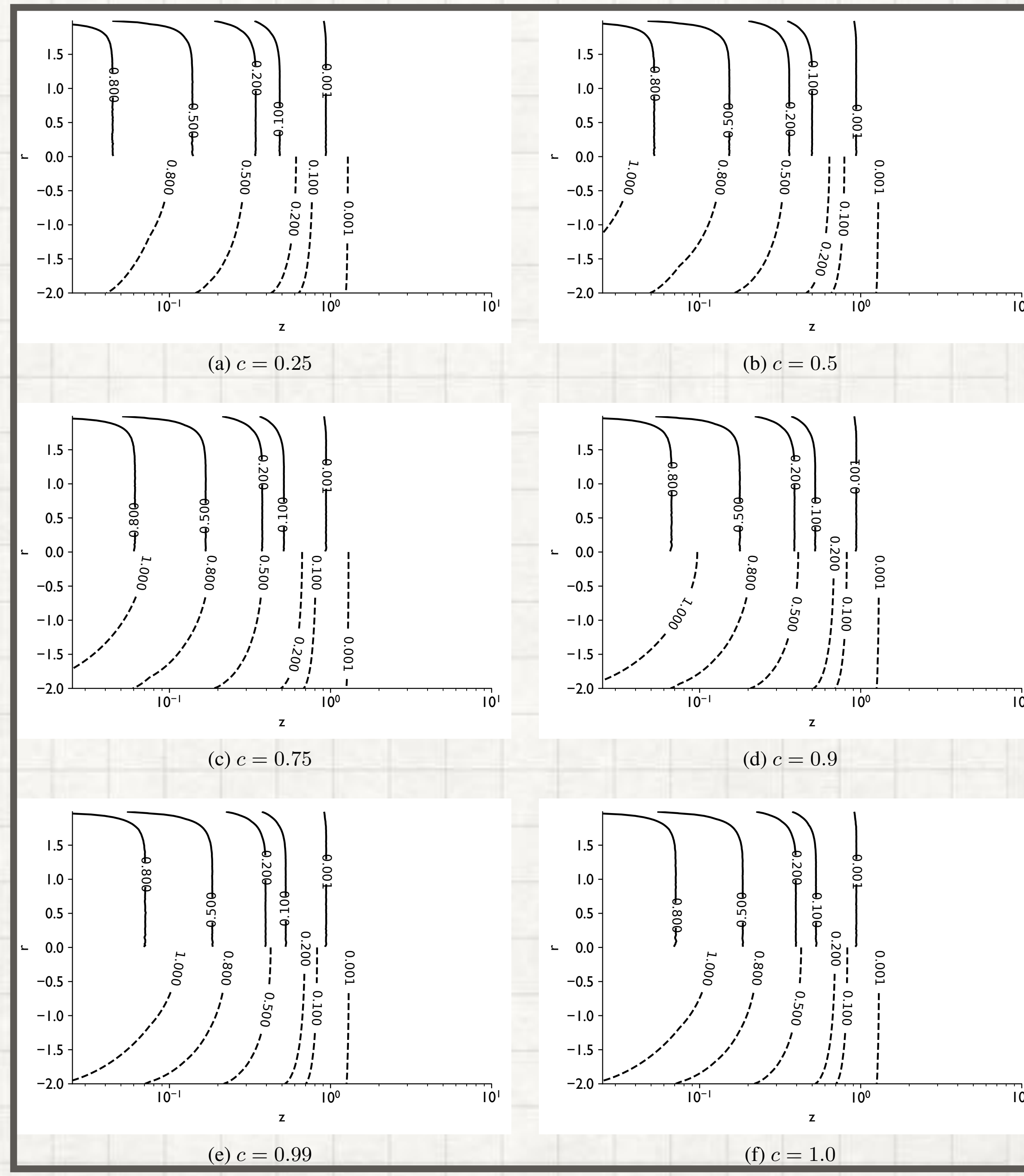


TIME-DEPENDENT SOLUTIONS HAVE THE OPPOSITE PROBLEM

3 Basis Functions Solutions At 1 Mean-Free Time Move Too Fast



Radius 6



Radius 2

Non-Constant Cross-Sections

- Beyond adding the coupling to the material temperature equation, there are additional complications to solving the 2-D Marshak wave problem in a cylinder.
- Spatially dependent cross-sections will cause create additional coupling between the angular fluxes.
- In principle, these can all be handled.
- An interesting question is the diffusion limit of this model.
- Scaling the equations so that scattering is large, and absorption, time-dependence, sources, *and the correction terms* are small, I get that the leading order solution satisfies

$$\frac{1}{v}\frac{\partial \phi_i^{(0)}}{\partial t} - \frac{\partial}{\partial z}\frac{1}{3\sigma_{\mathrm{t}}}\frac{\partial \phi_i^{(0)}}{\partial z} + \sigma_{\mathrm{a}}\phi_i^{(0)} + \frac{\pi}{4}\sum_{j=1}^{3} a_{ij}\phi_j^{(0)} = 2Q_i.$$