Improved Discrete Ordinates Solutions using Angular Filtering

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All numerical methods for deterministic transport calculations have problems when the angular variables are under resolved. The two most well know examples are ray effects in discrete ordinates (S_n) calculations and wave effects in spherical harmonics calculations (P_n). To date, most approaches to obtaining higher fidelity S_n solutions have been focused on quadrature sets that allow partial refinement and/or very high levels of refinement. In the past spherical harmonics methods have improved by the addition of closures (for example the popular M1 method). Recently, it was demonstrated [1] that a filtered spherical harmonics expansion could yield very accurate solutions to difficult transport problems.

The P_n method is formally a spectral method in the angular variable. As such, for smooth solutions it converges very quickly. In transport problems where the solution is non-smooth in angle, which is nearly every problem with a significant amount of streaming, Gibbs' oscillations appear in angle. These oscillations lead to the aforementioned wave effects and can also lead to negative scalar fluxes (a clearly non-physical result). The idea of a filter looks to decrease the strengths of these oscillations by reformulating the spherical harmonics expansion. Besides the standard moment-based derivation, the spherical harmonics expansion can be derived by minimizing the functional

$$\mathcal{J} = \int\limits_{4\pi} d\Omega \left(\psi(\Omega) - \sum_{l=0}^{N} \sum_{m=-l}^{l} Y_l^m(\Omega) a_l^m \right)^2 , \tag{1}$$

over the constants a_l^m, where the $Y_l^m(\Omega)$ are the spherical harmonics functions, ψ is the angular flux, and Ω is the angular variable (a point on the unit sphere). Note that this minimizes the integral of error squared, therefore it does not necessarily penalize large oscillations. To address this, one can add a penalty term:

$$\mathcal{J}_\alpha = \int\limits_{4\pi} d\Omega \left(I\psi(\Omega) - \hat{\psi}_N(\Omega) \right)^2 + \alpha \int\limits_{4\pi} d\Omega \left(\nabla_\Omega^2 \hat{\psi}_N(\Omega) \right)^2 , \tag{2}$$

where ∇_Ω is the gradient operator with respect to the angular variable, α is a filter strength and $\hat{\psi}_N$ is the modified expansion,

$$\hat{\psi}_N = \sum_{l=0}^{N} \sum_{m=-l}^{l} Y_l^m(\Omega) b_l^m .$$

Using the properties of the spherical harmonics functions, one can show that

$$b_l^m = \frac{a_l^m}{1 + \alpha l^2 (l+1)^2} . \tag{3}$$

Therefore, for a given expansion we can transform it to a filtered expansion.

To apply the method to the discrete ordinates equations we need a means of expressing the solution from a S_n calculation in a spherical harmonics expansion and a means to convert back to the S_n unknowns. We also must be able to do this in such a way that when there is no filter applied, we obtain the origin S_n unknowns (i.e., the transformation is invertible). To do this we use Galerkin quadrature sets [2], which have this exact property. The procedure for obtained filtered S_n solutions that we use looks to have the standard S_n solution relax to a filtered solution by adding an anisotropic scattering term given by

$$\sigma_{\mathrm{s}lm} = \frac{1}{1 + \alpha l^2 (l+1)^2} - 1 = -\frac{\alpha l^2 (l+1)^2}{1 + \alpha l^2 (l+1)^2} .$$

The equations are solved using the standard source iteration technique. To test the method we solve a similar problem to one from a previous paper on ray-effect mitigation techniques [3]. In this problem $\sigma_{\mathrm{t}} = 0.75$, $\sigma_{\mathrm{s}} = 0.5$, the domain is a 4 by 4 square with a source of strength 0.25 of dimensions 0.5 by 0.5 in the middle of the geometry. We look at the solution along the top quarter of the domain ($y = 4$ and $x \in [2, 4]$). As in previous work [1] we set the value of α as

$$\alpha = \frac{\omega}{n^2} \frac{1}{(\sigma_{\mathrm{s}} L + n)^2}.$$

where n is the order of the S_n method, L is a length scale we set to 1, and ω is a free-parameter that allows the user to control the strength of the filter. The results in Figure 1 show that the inclusion of the filter ($\omega > 0$) does dampen ray-effects, but too large of value of ω causes the solution to be too low in the corner of the problem.

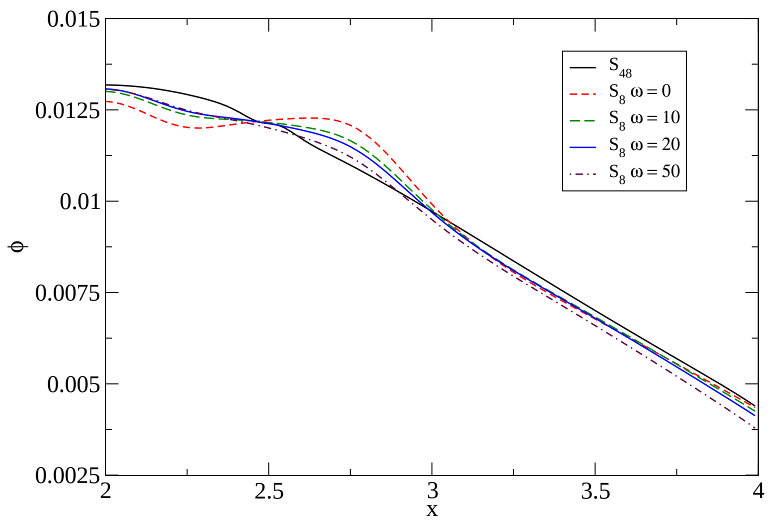

Figure 1: Scalar flux solution to test problem along top of domain from $x = 2$ to $x = 4$.

References

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