P_2-Equivalent form of the SP_2 Equations

Including boundary and interface conditions

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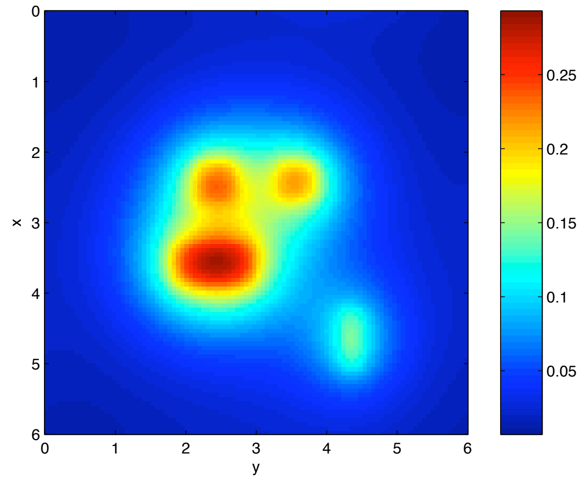
SP_n Equations

- Not news to anyone here but…
- The SP_n equations are a simplified form of the full spherical harmonics (P_n) equations for the linear transport equation.
  - *The SP_n equations have n+1 angular unknowns in first-order form*
  - *The P_n equations have $(n+1)^2$ unknowns*
- This large reduction of unknowns comes with a price
  - *The SP_n solution does not necessarily converge to the transport solution as n goes to ∞*
  - *As a result there is a order n that gives the optimal solution*
    - Another way of saying that: the error between the SP_n solution and the transport solution is lowest for some finite n.
- A colloquial rule of thumb: "Where diffusion is ok, SP_n is better. Where diffusion is bad, SP_n is worse."

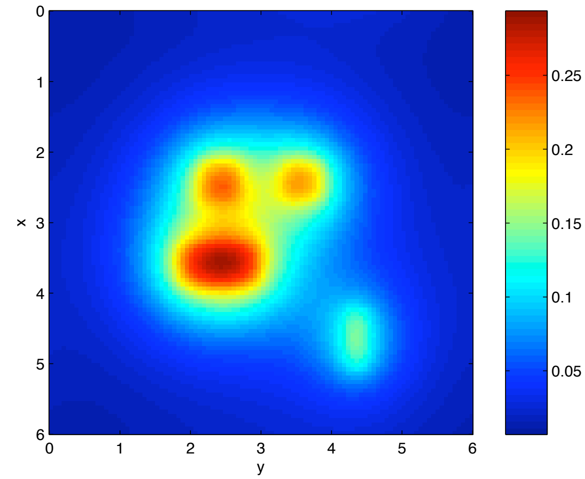
What do the SP_n equations represent?

- The SP_n equations were originally derived by Gelbard in 1960 by
  - *Taking the 1-D P_n equations*
  - *Replacing 1-D spatial derivatives*
    - With a gradient in the odd order equations
    - With a divergence in the even order equations
  - *Interpreting odd order unknowns as vectors and even order unknowns as scalars*
- Despite this ad hoc "derivation" the SP_n equations have the property that
  - *For an infinite medium, the SP_n solution is equivalent to the P_n solution provided that the total cross-section is constant and sources are isotropic*
  - *This fact has been used to solve some problems to high accuracy.*
    - Gelbard used it to compute the leakage from a cylinder by surrounding the cylinder with a pure absorber.

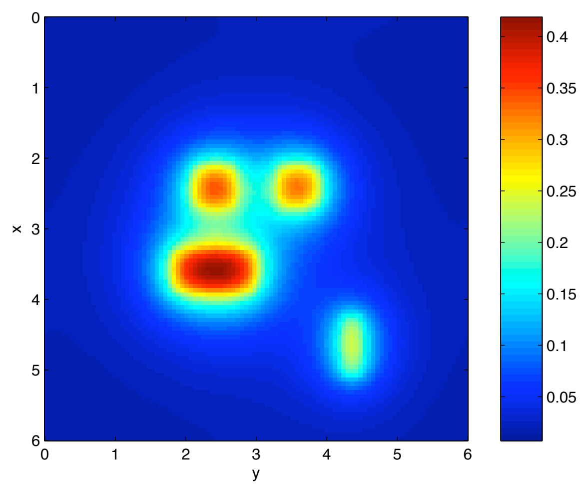
Example: Uniform medium with local sources



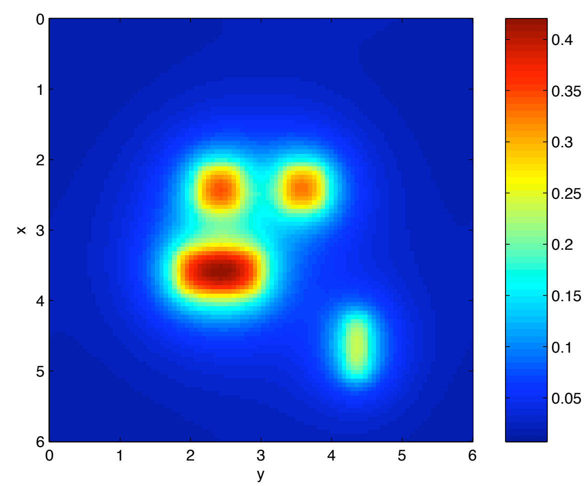
(a) P_1



(b) SP_1

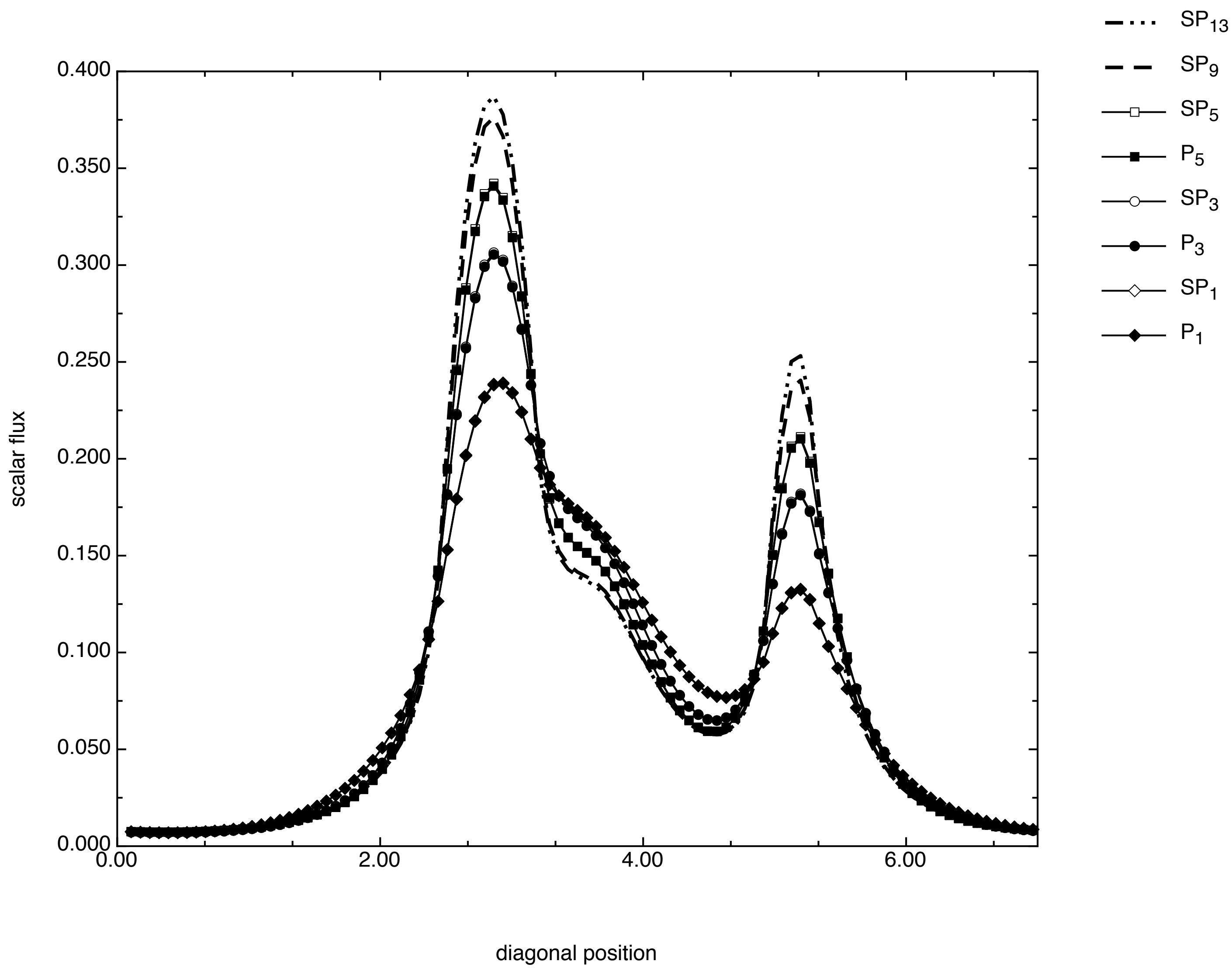


(c) P_5



(d) SP_5

Example: Uniform medium with local sources



What do the SP_n equations represent?

- The SP_n equations aren't just some ad hoc equations that happen to be correct in some limits.
- The SP_n equations can also be derived via an asymptotic expansion
  - *One approach does a similar expansion to that used to derive the diffusion limit (Larsen, Morel, and Miller)*
    - That is, scattering is large and absorption and sources are small.
    - This explains the rule of thumb expressed earlier
  - *The other approach expands the dependence of the solution in 2 of 3 spatial directions (Pomraning)*
    - This derivation shows that if the transport solution is "locally 1-D" the SPn solution will be accurate.
- Variational derivations also exist for the
  - *SP2 equations (Larsen and Tomasevic)*
  - *SP3 equations (Larsen and Brantley)*

Why look for P_n Equilavent Forms of SP_n

- We know that in a uniform, infinite medium the P_n and SP_n equations give the same scalar flux.
- Therefore, if we think of a heterogeneous materials in a finite problem as a patchwork of uniform media
  - *The only difference between the P_n and SP_n solutions comes down*
    - Material interface conditions
    - Boundary conditions
- If we can express the P_n conditions at boundaries and interfaces using SP_n unknowns
  - *We can derive an SP_n system that is equivalent to the P_n system in the scalar flux solution.*
- Of course, this might not be possible.
- At low order, one might have hope because the SP_n and P_n unknowns are the same through first-order
  - *The scalar flux and the current unknowns are the same in both systems.*

P_3-Equivalent SP_3 Equations have been claimed in the past.

- In a terse ANS transactions paper in 1970, Selengut claimed to have derived a form of the P_3 approximation
  - *That could be expressed entirely in terms of the SP_3 unknowns in second-order form*
  - *This approximation included interface conditions.*
  - *No boundary conditions though.*
- The brevity of the derivation makes reproducing the result difficult.
- No numerical results were presented
  - *I'm not aware of any attempts to solve these equations.*
- Some more recent analysis suggests that the solution used to construct these SP_3 equations might not be the most general solution.

Selengut, D. S. (1970). A new form of the P3 approximation. Trans. Am. Nucl. Soc. 13:625.

- ◆ **This is the entirety of the technical content in Selengut's ANS transactions.**

The neutron angular distribution can be written

$$\psi(\mathbf{r},\mathbf{\Omega}) = \left(1 - \frac{1}{\Sigma}\ \mathbf{\Omega}\cdot\nabla\right)\psi_e(\mathbf{r},\mathbf{\Omega})\ , \qquad (1)$$

where the even part satisfies the second-order Boltzmann equation[3,4]

$$-\mathbf{\Omega}\cdot\nabla\frac{1}{\Sigma}\mathbf{\Omega}\cdot\nabla\psi_e + \Sigma\psi_e = \frac{1}{4\pi}\Sigma_s\int d\Omega\psi_e + \frac{1}{4\pi}S\ . \qquad (2)$$

The P_3 approximation follows from setting

$$\psi_e(\mathbf{r},\mathbf{\Omega}) \simeq \frac{1}{4\pi}\phi(\mathbf{r}) + \frac{15}{8\pi}\sum_{ij}\rho_{ij}(\mathbf{r})\Omega_i\Omega_j\ , \qquad (3)$$

where ρ_{ij} is a traceless second-rank Cartesian tensor related to the second moment of ψ, and Ω_i is the i'th component of a unit vector along the neutron velocity.

Requiring conservation of the zero'th and second moments of Eq. (2) yields the tensor P_3 equations

$$\rho_{ij} = \frac{15}{8\pi}\sum_{klmn}\omega_{ijklmn}D_{kl}\rho_{mn} + \frac{1}{4\pi}\sum_{kl}\omega_{ijkl}D_{kl}\phi + \frac{1}{3\Sigma}\delta_{ij}(S - \Sigma_a\phi)\ , \qquad (4)$$

where $\omega_{ijk\ldots} = \int d\Omega\Omega_i\Omega_j\Omega_k\ldots$ is a family of isotropic tensors which can be evaluated explicitly and $D_{ij} = (\Sigma)^{-1}\partial/(\partial x_i)(\Sigma)^{-1}\partial/(\partial x_j)$. Contracting Eq. (4) with the operator D_{ij}, and using the trace of Eq. (4), one obtains

$$\frac{3}{35\Sigma^3}\nabla^4\phi + \frac{11}{21\Sigma}\nabla^2(S - \Sigma_a\phi) - \frac{1}{3\Sigma}\nabla^2\phi = S - \Sigma_a\phi\ , \qquad (5)$$

the known 4'th-order differential equation for the flux, which holds for both the complete and simplified forms of P_3 theory.

Applying Gauss' theorem to Eq. (2) implies that $\mathbf{n}\cdot\mathbf{\Omega}(\Sigma)^{-1}\nabla\cdot\mathbf{\Omega}\psi_e$ and $(\mathbf{n}\cdot\mathbf{\Omega})^2\psi_e$ must be continuous at interfaces between media, where $\mathbf{n}$ is the unit normal to the surface. This leads to the continuity of

$$\phi\ ,\quad \sum_{ij}n_i\rho_{ij}n_j\ ,\quad \frac{1}{3\Sigma}\mathbf{n}\cdot\nabla\phi + \sum_{ij}\frac{1}{\Sigma}n_i\nabla_j\rho_{ij}\ ,$$

and

$$\equiv \frac{2}{15\Sigma}\mathbf{n}\cdot\nabla\phi + \frac{1}{\Sigma}(\mathbf{n}\cdot\nabla)\sum_{ij}n_i\rho_{ij}n_j\ . \qquad (6)$$

To evaluate these interface conditions in terms of the flux, we can write the solution to Eq. (4) as

$$\rho_{ij} = \delta_{ij}\left[\frac{11}{42\Sigma}(S - \Sigma_a\phi) + \frac{3}{70\Sigma^2}\nabla^2\phi\right] + D_{ij}\left[\frac{2}{15}\phi - \frac{121}{294\Sigma}(S - \Sigma_a\phi) - \frac{33}{490\Sigma^2}\nabla^2\phi\right]\ . \qquad (7)$$

The flux Eq. (5) can now be solved using a coupled diffusion-theory code, after which the angular distribution is given explicitly by Eqs. (1), (3), and (7).

A convenient way to carry this out is to introduce the "pseudoflux" θ to obtain

$$-\frac{9}{55\Sigma}\nabla^2\phi - \frac{7}{11}\Sigma\theta + \Sigma_a\phi = S$$

$$+\frac{9}{28\Sigma}\nabla^2\theta - \frac{5}{4}\Sigma\theta + \Sigma_a\phi = S \qquad (8)$$

subject to the continuity at interfaces of

$$\phi\ ,\quad \frac{1}{\Sigma}\mathbf{n}\cdot\nabla(\phi + \theta)\ ,\quad \theta - \frac{1}{\Sigma^2}(\mathbf{n}\times\nabla)^2\left(\frac{28}{55}\phi + \theta\right)\ ,$$

and

$$\frac{1}{\Sigma}\mathbf{n}\cdot\nabla\theta + \frac{1}{\Sigma^2}(\mathbf{n}\times\nabla)^2\frac{1}{\Sigma}\mathbf{n}\cdot\nabla\left(\frac{2}{3}\phi + \frac{55}{42}\theta\right)\ . \qquad (9)$$

The cross-product terms are missing in the simplified P_3 approximation; for the case of spherical or cylindrical

The linear transport equation and SP_2 equations

- We'll begin the derivation of a P_2-equivalent form of the SP_2 equations with a steady, one-speed transport equation with isotropic scattering

$$(\Omega \cdot \nabla + \sigma_{\mathrm{t}})\,\psi = \frac{1}{4\pi}\left(\sigma_{\mathrm{s}}\phi + Q\right)$$

- In this situation the SP_2 equations are

$$\sigma_{\mathrm{a}}\phi_0 + \nabla \cdot \vec{\phi}_1 = Q,$$

$$\sigma_{\mathrm{t}}\vec{\phi}_1 + \frac{1}{3}\nabla\phi_0 + \frac{2}{3}\nabla\phi_2 = 0,$$

$$\sigma_{\mathrm{t}}\phi_2 + \frac{2}{5}\nabla \cdot \vec{\phi}_1 = 0,$$

The P_2 Equations in 2-D Cartesian geometry

- If we restrict our system to 2-D x-z geometry, the full P2 equations are

$$\sigma_{\mathrm{a}}\psi_{00} + \frac{\partial}{\partial z}\psi_{10} + \frac{\partial}{\partial x}\psi_{11} = Q,$$

$$\sigma_{\mathrm{t}}\psi_{10} + \frac{\partial}{\partial z}\left(\frac{1}{3}\psi_{00} + \frac{2}{3}\psi_{20}\right) + \frac{1}{3}\frac{\partial}{\partial x}\psi_{21} = 0,$$

$$\sigma_{\mathrm{t}}\psi_{11} + \frac{\partial}{\partial x}\left(\frac{1}{3}\psi_{00} - \frac{1}{3}\psi_{20} + \frac{1}{6}\psi_{22}\right) + \frac{1}{3}\frac{\partial}{\partial z}\psi_{21} = 0,$$

$$\sigma_{\mathrm{t}}\psi_{20} + \frac{2}{5}\frac{\partial}{\partial z}\psi_{10} - \frac{1}{5}\frac{\partial}{\partial x}\psi_{11} = 0,$$

$$\sigma_{\mathrm{t}}\psi_{21} + \frac{3}{5}\frac{\partial}{\partial z}\psi_{11} + \frac{3}{5}\frac{\partial}{\partial x}\psi_{10} = 0,$$

$$\sigma_{\mathrm{t}}\psi_{22} + \frac{6}{5}\frac{\partial}{\partial x}\psi_{11} = 0.$$

- The moments are defined as $\psi_{lm} = \int_{4\pi} Y_{lm}(\Omega)\psi(\Omega)\, d\Omega$

Simplifying these equations

- The first step to writing the P2 equations in SP2 form defines

$$\phi_2 = \psi_{20} + \frac{\psi_{22}}{2}$$

- Then we add the ψ_{20} equation to one-half times the ψ_{22} equation to get

$$\sigma_{\mathrm{t}}\phi_2 + \frac{2}{5}\frac{\partial}{\partial z}\psi_{10} + \frac{2}{5}\frac{\partial}{\partial x}\psi_{11} = 0,$$

- Upon defining the current to be $\vec{J} = (\psi_{11}, 0, \psi_{10})^t$, this equation becomes

$$\sigma_{\mathrm{t}}\phi_2 + \nabla\cdot\vec{J} = 0,$$

- This is exactly the last equation in the SP_2 system.

The ψ_{1m} equations are not so easily simplified.

- From the original equations we can add equations to make simplifications

$$\sigma_{\mathrm{a}}\psi_{00} + \frac{\partial}{\partial z}\psi_{10} + \frac{\partial}{\partial x}\psi_{11} = Q,$$

$$\sigma_{\mathrm{t}}\psi_{10} + \frac{\partial}{\partial z}\left(\frac{1}{3}\psi_{00} + \frac{2}{3}\psi_{20}\right) + \frac{1}{3}\frac{\partial}{\partial x}\psi_{21} = 0,$$

$$\sigma_{\mathrm{t}}\psi_{11} + \frac{\partial}{\partial x}\left(\frac{1}{3}\psi_{00} - \frac{1}{3}\psi_{20} + \frac{1}{6}\psi_{22}\right) + \frac{1}{3}\frac{\partial}{\partial z}\psi_{21} = 0,$$

$$\sigma_{\mathrm{t}}\psi_{20} + \frac{2}{5}\frac{\partial}{\partial z}\psi_{10} - \frac{1}{5}\frac{\partial}{\partial x}\psi_{11} = 0,$$

$$\sigma_{\mathrm{t}}\psi_{21} + \frac{3}{5}\frac{\partial}{\partial z}\psi_{11} + \frac{3}{5}\frac{\partial}{\partial x}\psi_{10} = 0,$$

$$\sigma_{\mathrm{t}}\psi_{22} + \frac{6}{5}\frac{\partial}{\partial x}\psi_{11} = 0.$$

The ψ_{1m} equations are not so easily simplified.

- We make the substitutions

$$-\frac{1}{3}\psi_{20} + \frac{1}{6}\psi_{22} = \frac{2}{3}\left(\phi_2 + \frac{3}{5\sigma_{\mathrm{t}}}\frac{\partial}{\partial z}\psi_{10}\right)$$

$$\frac{2}{3}\psi_{20} = \frac{2}{3}\phi_2 - \frac{1}{3}\psi_{22} = \frac{2}{3}\phi_2 + \frac{2}{5\sigma_{\mathrm{t}}}\frac{\partial}{\partial x}\psi_{11},$$

- This leads to the equations

$$\sigma_{\mathrm{t}}\psi_{10} + \frac{\partial}{\partial z}\left(\frac{1}{3}\psi_{00} + \frac{2}{3}\phi_2\right) = \frac{\partial}{\partial x}\frac{1}{5\sigma_{\mathrm{t}}}\frac{\partial}{\partial z}\psi_{11} + \frac{\partial}{\partial x}\frac{1}{5\sigma_{\mathrm{t}}}\frac{\partial}{\partial x}\psi_{10} - \frac{\partial}{\partial z}\frac{2}{5\sigma_{\mathrm{t}}}\frac{\partial}{\partial x}\psi_{11},$$

$$\sigma_{\mathrm{t}}\psi_{11} + \frac{\partial}{\partial x}\left(\frac{1}{3}\psi_{00} + \frac{2}{3}\phi_2\right) = \frac{\partial}{\partial z}\frac{1}{5\sigma_{\mathrm{t}}}\frac{\partial}{\partial z}\psi_{11} + \frac{\partial}{\partial z}\frac{1}{5\sigma_{\mathrm{t}}}\frac{\partial}{\partial x}\psi_{10} - \frac{\partial}{\partial x}\frac{2}{5\sigma_{\mathrm{t}}}\frac{\partial}{\partial z}\psi_{10}.$$

Putting this together leads to the SP2 equations, almost

- **At this point we have the standard SP2 equations with a strange right-hand side**

$$\sigma_{\mathrm{a}}\phi_0 + \nabla\cdot\vec{\phi}_1 = Q,$$

$$\sigma_{\mathrm{t}}\vec{\phi}_1 + \frac{1}{3}\nabla\phi_0 + \frac{2}{3}\nabla\phi_2 = \begin{pmatrix} \frac{\partial}{\partial z}\frac{1}{5\sigma_{\mathrm{t}}}\frac{\partial}{\partial z}\psi_{11} + \frac{\partial}{\partial z}\frac{1}{5\sigma_{\mathrm{t}}}\frac{\partial}{\partial x}\psi_{10} - \frac{\partial}{\partial x}\frac{2}{5\sigma_{\mathrm{t}}}\frac{\partial}{\partial z}\psi_{10} \\ 0 \\ \frac{\partial}{\partial x}\frac{1}{5\sigma_{\mathrm{t}}}\frac{\partial}{\partial z}\psi_{11} + \frac{\partial}{\partial x}\frac{1}{5\sigma_{\mathrm{t}}}\frac{\partial}{\partial x}\psi_{10} - \frac{\partial}{\partial z}\frac{2}{5\sigma_{\mathrm{t}}}\frac{\partial}{\partial x}\psi_{11} \end{pmatrix}$$

$$\sigma_{\mathrm{t}}\phi_2 + \frac{2}{5}\nabla\cdot\vec{\phi}_1 = 0,$$

with $\phi_0 = \psi_{00}$ and $\vec{J} = (\psi_{11}, 0, \psi_{10})^t$.

- **The RHS can be rewritten in terms of common vector calculus operators.**

Simplifying the RHS (Yes, that is the curl operator)

- First, we separate the RHS into a piece where σ_t is constant and a piece where σ_t is spatially varying using the product rule

$$\frac{1}{5}\begin{pmatrix} \frac{\partial}{\partial z}\frac{1}{\sigma_\mathrm{t}}\frac{\partial}{\partial z}\psi_{11} + \frac{\partial}{\partial z}\frac{1}{\sigma_\mathrm{t}}\frac{\partial}{\partial x}\psi_{10} - \frac{\partial}{\partial x}\frac{2}{\sigma_\mathrm{t}}\frac{\partial}{\partial z}\psi_{10} \\ 0 \\ \frac{\partial}{\partial x}\frac{1}{\sigma_\mathrm{t}}\frac{\partial}{\partial z}\psi_{11} + \frac{\partial}{\partial x}\frac{1}{\sigma_\mathrm{t}}\frac{\partial}{\partial x}\psi_{10} - \frac{\partial}{\partial z}\frac{2}{\sigma_\mathrm{t}}\frac{\partial}{\partial x}\psi_{11} \end{pmatrix} = \frac{1}{5\sigma_\mathrm{t}}\begin{pmatrix} \frac{\partial^2}{\partial z^2}\psi_{11} - \frac{\partial^2}{\partial x \partial z}\psi_{10} \\ 0 \\ \frac{\partial^2}{\partial x^2}\psi_{10} - \frac{\partial^2}{\partial x \partial z}\psi_{11} \end{pmatrix}$$

$$- \frac{2}{5}\begin{pmatrix} \left(\frac{\partial}{\partial z}\psi_{10}\right)\frac{\partial}{\partial x}\sigma_\mathrm{t}^{-1} \\ 0 \\ \left(\frac{\partial}{\partial x}\psi_{11}\right)\frac{\partial}{\partial z}\sigma_\mathrm{t}^{-1} \end{pmatrix} + \frac{1}{5}\begin{pmatrix} \left(\frac{\partial}{\partial z}\psi_{11} + \frac{\partial}{\partial x}\psi_{10}\right)\frac{\partial}{\partial z}\sigma_\mathrm{t}^{-1} \\ 0 \\ \left(\frac{\partial}{\partial x}\psi_{10} + \frac{\partial}{\partial z}\psi_{11}\right)\frac{\partial}{\partial x}\sigma_\mathrm{t}^{-1}. \end{pmatrix}$$

- We note that

$$\begin{pmatrix} \frac{\partial^2}{\partial z^2}\psi_{11} - \frac{\partial^2}{\partial x \partial z}\psi_{10} \\ 0 \\ \frac{\partial^2}{\partial x^2}\psi_{10} - \frac{\partial^2}{\partial x \partial z}\psi_{11} \end{pmatrix} = -\nabla \times \nabla \times \vec{\phi}_1.$$

The final SP2 equations are then

$$\sigma_{\mathrm{a}}\phi_0 + \nabla\cdot\vec{\phi}_1 = Q,$$

$$\sigma_{\mathrm{t}}\vec{\phi}_1 + \frac{1}{3}\nabla\phi_0 + \frac{2}{3}\nabla\phi_2 = -\frac{1}{5\sigma_{\mathrm{t}}}\nabla\times\nabla\times\vec{\phi}_1 - \frac{1}{5}\left(\nabla\times\vec{\phi}_1\right)\times\nabla\sigma_{\mathrm{t}}^{-1}$$
$$+\frac{2}{5}\begin{pmatrix}\left(\frac{\partial}{\partial z}\psi_{11}\right)\frac{\partial}{\partial z}\sigma_{\mathrm{t}}^{-1} - \left(\frac{\partial}{\partial z}\psi_{10}\right)\frac{\partial}{\partial x}\sigma_{\mathrm{t}}^{-1}\\ 0 \\ \left(\frac{\partial}{\partial x}\psi_{10}\right)\frac{\partial}{\partial x}\sigma_{\mathrm{t}}^{-1} - \left(\frac{\partial}{\partial x}\psi_{11}\right)\frac{\partial}{\partial z}\sigma_{\mathrm{t}}^{-1}\end{pmatrix},$$

$$\sigma_{\mathrm{t}}\phi_2 + \frac{2}{5}\nabla\cdot\vec{\phi}_1 = 0.$$

The SP2 equations with constant cross-section

- In the case when the total cross-section is constant, the right hand of the current equation simplifies, but does not go to zero:

$$\sigma_{\mathrm{t}}\vec{\phi}_1 + \frac{1}{3}\nabla\phi_0 + \frac{2}{3}\nabla\phi_2 = -\frac{1}{5\sigma_{\mathrm{t}}}\nabla\times\nabla\times\vec{\phi}_1,$$

- However, these terms do not influence the scalar flux
  - *Because the divergence of J does not contain them*

$$\nabla\cdot\vec{\phi}_1 = -\frac{1}{3\sigma_{\mathrm{t}}}\nabla^2\phi_0 - \frac{2}{3\sigma_{\mathrm{t}}}\nabla^2\phi_2,$$

  - *Therefore there are modes in the solution that are in the P2 equations but not the SP2 equations.*

The Differences between the SP2 equations and the P2 equations

- ***The extra terms in the reduced P2 equations, do not influence the scalar flux***
  - *Because the divergence of a curl is zero.*
- ***Equivalently, the null space of the P2 equations is not the same as that of the SP2 equations***
- ***The scalar flux in the SP2 equations is the same as the P2 equations***
- **The current is not necessarily the same in the two systems.**
- **Of course, unless boundary/interface conditions introduce these modes**
  - *These modes won't be created.*
- **Yet, if we solve the SP2 equations, w/o the extra terms but with the correct interface/boundary conditions**
  - *We would get the same scalar flux as the P2 equations*

Deriving material interface conditions

- To get interface conditions we write the P2 equations as a hyperbolic system

$$\left(\mathbf{A}_x \frac{\partial}{\partial x} + \mathbf{A}_z \frac{\partial}{\partial z} + \sigma_{\mathrm{t}}\right) \vec{\psi} = \delta_{l0}\delta_{m0}\left(\sigma_{\mathrm{s}}\phi_0 + Q\right)$$

- We can derive interface conditions by hypothesizing an interface in either the *x* or *z* directions
  - *And then diagonalizing the appropriate Jacobian*
  - *To find the waves that move in each direction*
- These interface conditions can then be expressed in terms of the SP2 unknowns.
- This approach to obtaining boundary conditions will yield Mark-type boundary conditions

Deriving Interface Conditions

- The eigenvalues for both the Jacobians are

$$\left(-\sqrt{\frac{3}{5}}, \quad \sqrt{\frac{3}{5}}, \quad -\frac{1}{\sqrt{5}}, \quad \frac{1}{\sqrt{5}}, \quad 0, \quad 0 \right)$$

  - *The zero eigenvalues means that there are two waves that do not move for each direction*
  - *These zero-eigenvalues can cause problems in numerical calculations*
  - *These are also a reason why even-over Pn expansions are generally avoided*

- From the eigenvectors we get that across an interface in the *z* direction, the following are continuous

$$\left(\phi + \frac{4}{5}\psi_{20}, \psi_{10}, \psi_{11}, \psi_{21}\right)$$

- In the *x* direction, these are continuous

$$\left(\phi - \frac{1}{5}\psi_{20} + \frac{6}{5}\psi_{22}, \psi_{10}, \psi_{11}, \psi_{21}\right)$$

- It's not clear how to enforce these conditions using just ϕ and ϕ_2

Summary

- **Using some simple manipulations the P2 equations in 2-D geometry can be written as an SP2 system**
  - *With some extra terms that don't influence the scalar flux*
  - *Except through boundary and interface conditions*
- **These equations demonstrate that even though the scalar flux between the two equations is consistent in a uniform media**
  - *The solution for the current is, in general, different.*
- **What about 3-D?**
  - *I've not had any luck getting a similar manipulation to work in 3-D for the P2 equations*
- **P3 or higher order?**
  - *Same story; partially due to the fact that in 2-D P3 has 4 more unknowns than P2*