Fully Implicit Filtered P_N for High-Energy Density Thermal Radiation Transport

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Motivation

- All deterministic methods to solve the transport equation have flaws
- The PN equations produce a rotationally-invariant solution and therefore is **immune to ray effects** (good for diffusive problems)
- BUT they can also create **unphysical oscillations** and **negativity** in the solution
- While it is often crucial to have positivity in numerous applications (radiative transfer,...)
- Idea: **damp** the large derivatives in angle

Fully-Implicit P_N Equations

- Transport equation:

$$\frac{1}{c}\partial_t\psi + \boldsymbol{\Omega}\cdot\boldsymbol{\nabla}\psi + \sigma_t\psi = \sigma_s\phi + Q$$

- Spherical Harmonics expansion:

$$\psi(\mathbf{r},\boldsymbol{\Omega}) \approx \sum_{l=0}^{N}\sum_{m=-l}^{l}\psi_l^m(\mathbf{r})Y_l^m(\boldsymbol{\Omega})$$

- Quasi steady-state formulation of the transport equation:

$$A_x\partial_x\psi^{n+1} + A_y\partial_y\psi^{n+1} + A_z\partial_z\psi^{n+1} + \sigma_t^*\psi^{n+1} = \sigma_s\phi^{n+1} + Q^{n,*}$$

- Linear system solved using a Krylov solver (PETSc):

$$L\psi + C\psi = Q^*$$

Filtering: McClarren/Hauck approach

- After each time step:

$$\psi_l^m \longleftarrow \frac{\psi_l^m}{1+\alpha l^2(l+1)^2}$$

Where:

$$\alpha = \frac{\omega}{N^2(\sigma_t L + N)^2}$$

ω: filter strength
L: characteristic length

Filtering: Radice/Abdikamalov approach

- Matrix form:

$$L\psi + (C+\Delta)\psi = Q^*$$

Where:

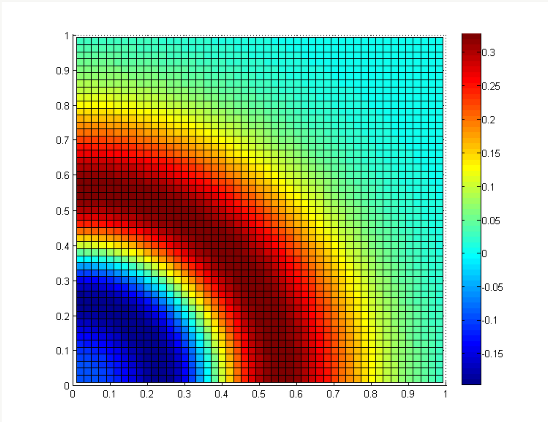
$$\Delta \equiv -\beta\log\sigma_{\text{filterType}}\left(\frac{l}{N+1}\right)$$

And:

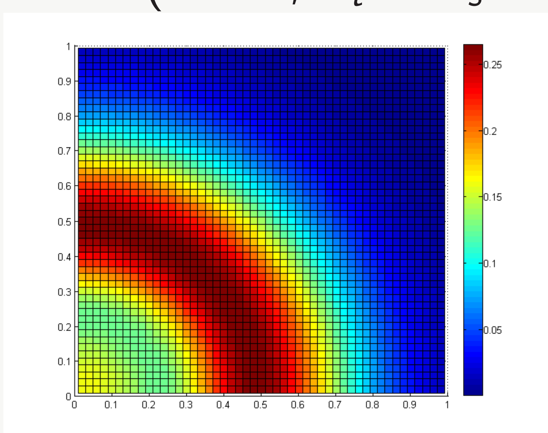
$$\sigma_{\text{filterType}} = \begin{cases} \sigma_{\text{Lanczos}}(\eta) = \dfrac{\sin\eta}{\eta} \\ \sigma_{\text{SSpline}}(\eta) = \dfrac{1}{1+\eta^4} \end{cases}$$

Effects of the filtering

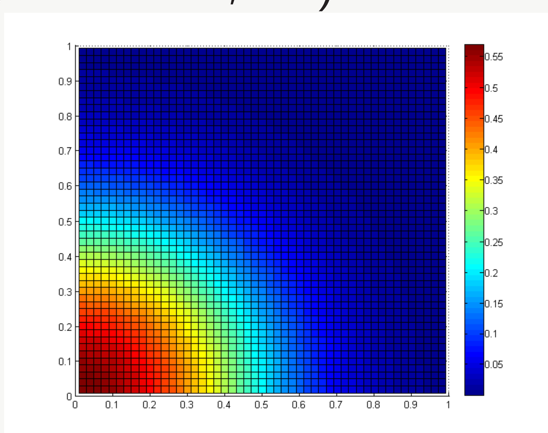
- Pulsed line source after $10\Delta t$ ($c=1$, $\sigma_t=\sigma_s=1$, $\Delta t=0.1$, P1)



$\omega = 0$



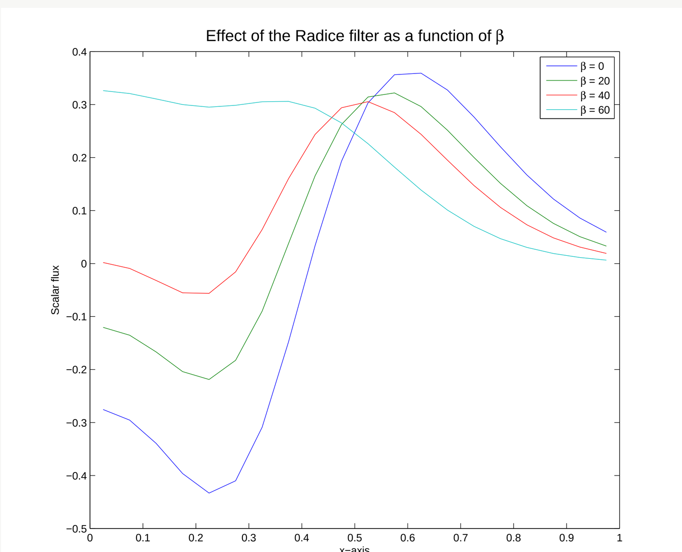
$\omega = 0.2$



$\omega = 1.0$

Properties of the filters:

- do not change the 0th moment (particle conservation)
- vanish as $N \to \infty$
- preserve the equilibrium diffusion limit
- preserve the rotational invariance

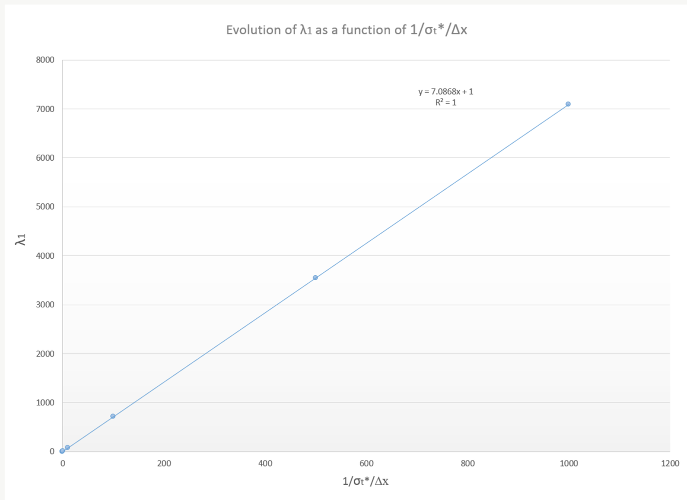


Eigenspectrum of $L + C$

Properties of the filters:

- All the eigenvalues are such that $\Re\lambda \geq 1$
- Smallest eigenvalue $\equiv \lambda_{\min} = 1$
- Largest eigenvalue $\equiv \lambda_1$

$$\lambda_1 = 1 + \eta\frac{1}{\Delta x\,\sigma_t^*}$$



Impact on the eigenspectrum in void

$\frac{c\Delta t}{\Delta x}$	β	λ_1	$\lambda_{\min}$	Γ
1	0	8.0868	1.0000	100%
1	20	8.0842	1.0840	91.1%
1	60	8.0788	1.1591	84.2%
1	100	8.0727	1.0981	89.6%
10	0	71.868	1.0000	100%
10	20	71.842	1.8404	53.7%
10	60	71.788	2.5911	37.7%
10	100	71.727	1.9807	49.7%

$$\gamma \equiv \frac{|\lambda_1 - \lambda_{\min}|}{|\lambda_{\min}|}$$

$$\Gamma \equiv \frac{\gamma}{\gamma_{\text{unfiltered}}}$$

Thermal Radiation Transfer

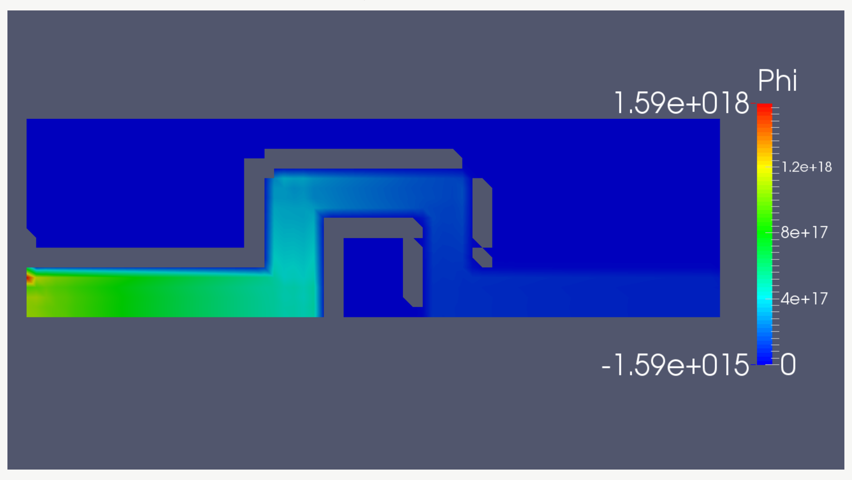
- Add temperature equation:

$$\begin{cases} \dfrac{1}{c}\partial_t I + \boldsymbol{\Omega}\cdot\boldsymbol{\nabla}I + \sigma_t I = \sigma_a B(T) + \sigma_s\varphi + Q \\ C_v\,\partial_t T = \sigma_a\big(\varphi - 4\pi B(T)\big) + Q_e \end{cases}$$

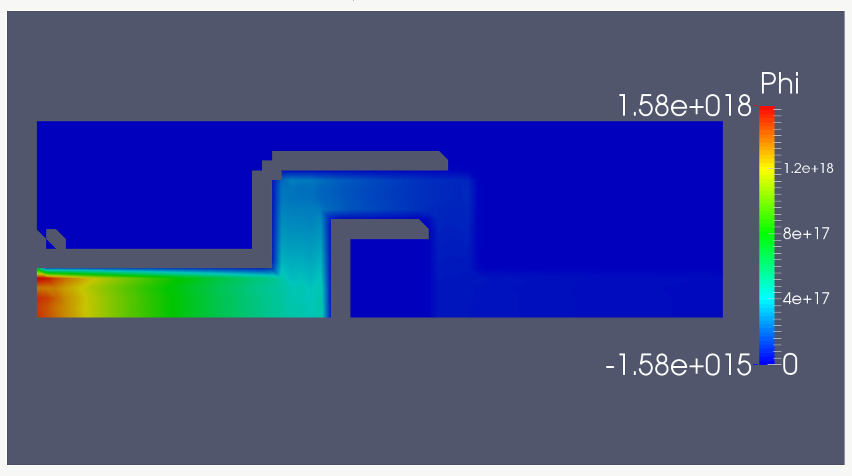
Crooked Pipe Test Problem

- Scalar flux after $1000\Delta t$ ($\Delta t = 4\times10^{-12}$ s, P1)

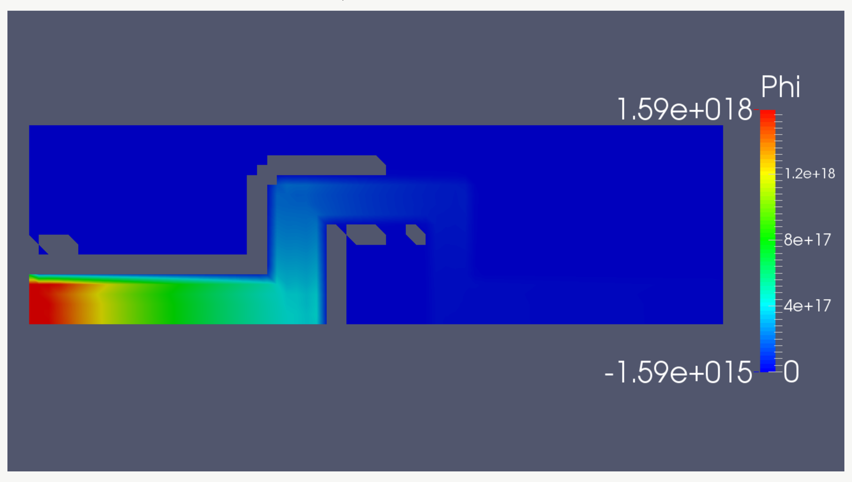
$\beta = 0$



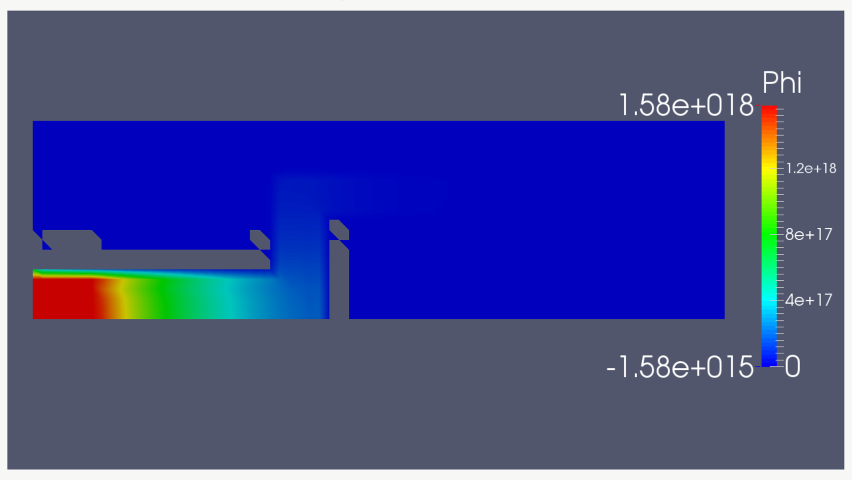
$\beta = 50$



$\beta = 100$



$\beta = 500$



- Helps smooth the solution
- The strength of the filter should not be too large

Conclusions

- Filtered PN reduces the **negativity** and **oscillations** of the solution
- Filtered PN **improves the conditioning** of the linear system
- Optimum filter strength in void depending on $\frac{c\Delta t}{\Delta x}$
- Reduces the speed of the waves

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